Analysis of Students' Errors in Solving the Least Common Multiple and Greatest Common Divisor Word Problems

Ana Puji Lestari¹*, Inti Nahdataeni S.²

¹ Universitas Negeri Malang, Jurusan Matematika, Prodi S2 Pendidikan Matematika
² SMP IT Al-Fahmi Palu

*Corresponding author: anapujilestari.1903118@students.um.ac.id, Palu, Sulawesi Tengah, 94111, Indonesia.

Abstract

How to cite this article:
doi: http://dx.doi.org/10.24235/eduma.v11i2.9324

Article history:
Received: 11 12, 21
Accepted: 12 01, 22
Published: 12, 22

Copyright © 2021
EduMa: Mathematics Education Learning and Teaching under the Creative Commons Attribution 4.0 International License.

Keywords:
Newman error analysis; least common multiple; greatest common divisor; word problem

Students' ability to solve word problems has an essential role in learning mathematics. However, some junior high school students still have difficulty in solving word problems. The difficulties experienced by students cause errors when solving mathematical word problems. This qualitative study aims to analyze students' errors in solving the Least Common Multiple and Greatest Common Divisor word problems. Fifty-nine students of Integrated Islamic Junior High School Al-Fahmi in Palu were involved in this study. Two students were selected to participate in the interview process. The interview was conducted to obtain in-depth information about how students solve word problems and the types of errors experienced by students. Students' errors in solving word problems were analyzed based on Newman's theory. The errors experienced by students are comprehension errors, transformation errors, process skills errors, and encoding errors.
INTRODUCTION

Word problem has an essential role in mathematics education today (Boonen et al., 2016). According to (Csíkos & Szitányi, 2020), word problems need to be taught to students starting from the elementary level in learning mathematics by presenting non-routine questions. The ability to solve word problems has proven to be very helpful for students to connect mathematics with real-life (Wong & Ho, 2017). The importance of word problems also encourages scholars to investigate deeper through research in mathematics education.

Research related to word problems has been carried out by several world researchers, such as the research conducted by (Csíkos & Szitányi, 2020), which examined teachers' pedagogical content in teaching word problem-solving. Meanwhile (González-Castro et al., 2016) examines integrated dynamic representation (IDR) to improve early primary mathematics competencies and word problem-solving skills. In addition, the influence of the pre-algebraic reasoning component on the performance of students experiencing mathematical difficulties in solving word problems was investigated by (Powell et al., 2020). Another study was also conducted by (Van Dooren et al., 2019), which examined increasing the ability to solve word problems using humor. These studies prove that the study of mathematical word problems has received the attention of several world scholars.

Although scientists have widely researched word problems, it turns out that some students still have difficulty solving word problems (Powell et al., 2020; Verschaffel et al., 2020). For example, a study conducted by (Powell et al., 2020) stated that elementary level students often have difficulty solving word problems. Some students still experience errors in formulating equations, schemes, or diagrams, causing students difficulty in mathematical modeling (Jupri & Drijvers, 2016). (Boonen et al., 2016) revealed that word problems become increasingly challenging for students who have difficulty learning mathematics when several steps or operations are required to solve word problems.

The difficulties experienced by students in solving word problems cause errors that occur during the process of solving a word problem. This error indicates that some students have difficulty solving mathematical word problems that integrate real-life problems (Han et al., 2016). Student errors in solving word problems can occur either in errors about prerequisite knowledge or newly constructed knowledge (Granberg, 2016).

Several researchers have carried out studies related to error analysis (Granberg, 2016; Hu et al., 2021; Rushton, 2018). The study results (Granberg, 2016) show that students experience errors in prior knowledge and newly constructed knowledge. Meanwhile, research conducted by (Hu et al., 2021) examined the interpretation and responses of Algebra teachers to students’ errors in solving quadratic equations. (Rushton, 2018) revealed that the use of error analysis is beneficial for teachers and students in the mathematics learning process.

The researcher's interview with the mathematics teacher at Integrated Islamic of Junior High School Al-Fahmi Palu shows that some students still often experience errors in solving word problems related to the Least Common Multiple (LCM) and Greatest Common Divisor (GCD). Students tend to work on the word LCM using the GCD method and vice versa. Therefore, researchers need to explore students’ errors in solving the LCM and GCD word problems.
Student errors in solving word problems can be analyzed based on Newman's theory. According to (Khalo et al., 2015), Newman's error analysis provides a framework for considering the underlying reasons for the difficulties experienced by students, for example, word problems and processes that help teachers determine where comprehension error occurs. Mathematical problem-solving analysis based on Newman's theory has the potential to focus students' attention on the structure of mathematical problems and facilitate their metacognitive awareness and subsequent problem-solving abilities (Reid O'Connor & Norton, 2020).

Newman's theory classifies the types of student errors based on reading errors, comprehension errors, transformation errors, processing skills errors, and encoding errors (Clements, 1980). According to (Clements, 1980), reading errors occur when students experience errors in reading important information contained in questions that result in students not using the information to solve problems. Comprehension errors occur when students read the questions well but do not understand the information in the questions. For example, students are unable to identify the information that is known and asked in the questions. Transformation errors occur when students fail to change problems in mathematical models such as equations, pictures, graphs, and tables. Process skill errors occur because students experience errors in the calculation process. Encoding errors occur when students cannot write answers correctly and do not write conclusions.

Several researchers have conducted studies on error analysis based on Newman's theory (Agustiani, 2021; Fitriani et al., 2018; Singh et al., 2010; Trapsilasiwi et al., 2021). A study on the analysis of student errors in solving the application problem of sequences and series using the Newman procedure has been carried out by (Agustiani, 2021). (Fitriani et al., 2018) examined the analysis of student errors in solving algebraic function problems. Meanwhile, (Singh et al., 2010) examined the analysis of elementary students' errors on mathematics written assignments. (Trapsilasiwi et al., 2021) examined the error analysis of introverted students based on Newman's theory in solving arithmetic sequences and series problems.

Based on the background description, it is necessary to conduct an in-depth study of student errors in learning mathematics, especially in solving word problems. This study will analyze the errors of junior high school students in solving word problems on the LCM and GCD topics based on Newman's theory.

**METHODS**

A qualitative approach was used in this study (Creswell & Creswell, 2017). This study aimed to analyze student errors in solving the LCM and GCD word problem based on Newman's theory. Fifty-nine grade VII students of Integrated Islamic Junior High School Al-Fahmi in Palu were involved in this study. The research instrument used a problem-solving test in the form of word problems and interview guidelines. The interview guide was adapted from (White, 2005). Two students were selected as participants in this study to participate in the interview process. Interviews were conducted to obtain in-depth information about students' errors in solving the LCM and GCD word problems. The following is a word problem about LCM and GCD presented to students:

1. Nadia went to a stationery shop to buy 84 pencils, 60 pens, and 96 markers. All stationery will be distributed to her friends, packaged in packages so that no
stationery is leftover. Nadia wants her friends to have the same amount of each type of stationery. What is the maximum number of packages that Nadia can create?

2. Radit, Fikar, and Budi will take an online mathematics course. They need credits to support the implementation of online courses. Radit buys credit every 15 days, Fikar buys credit every 30 days, and Budi buys credit every 45 days. If the three of them buy credit together on April 30, 2020, when will they buy credit together again?

Findings

This study describes the errors experienced by students in solving the LCM and GCD word problems. The errors experienced by students will be analyzed based on Newman’s theory. A summary of the results of students' work in solving the LCM and GCD word problems is presented in table 1 below.

Table 1. Summary of Students Performance Results on Questions Number 1 and 2

<table>
<thead>
<tr>
<th>Question Numbers</th>
<th>Correct</th>
<th>Incorrect</th>
<th>Unattempted</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26</td>
<td>31</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
<td>40</td>
<td>6</td>
</tr>
</tbody>
</table>

Regarding the summary of student performance on question number 1, it shows that twenty-six students (44.1%) answered incorrectly, and thirty-one students (52.5%) answered correctly. Two students (3.4%) did not answer. The errors made by students on question number 1 showed varied results. A student error in determining the concept, namely the GCD problem, was solved with the LCM concept. Another error is that students failed in determining conclusions and errors in determining the GCD value.

In question number 2, there are 22% (13 students) answered correctly, 67.8% (40 students) answered incorrectly, and 10.2% (6 students) did not answer the question. The errors experienced by students in solving problem number 2 are students experiencing errors in determining conclusions and students being failed in determining the value of the LCM. The following is a representation of the errors experienced by students with the initials AF and MF.

Analysis of the results of AF’s work

Based on the results of AF’s work, question number 1 related to the GCD problem can be done correctly, while number 2 related to the LCM problem is carried out using the GCD method. The AF views the problem of number 2 as a GCD problem. AF’s work shows that in solving problem number 2, AF first writes down the known and asked information in the problem. Then, AF creates a factor tree of the numbers 15, 30, and 45. Figure 1. in the following is the results of AF’s work on problem number 2.
Figure 1
AF’s work on number 2

The factor tree procedure of 45, 30, and 15 is written by AF correctly. Next, AF writes the factorization of the prime numbers of 45, 30, and 15, namely

\[ 15 = 3 \times 5; \]
\[ 30 = 2 \times 3 \times 5; \]
\[ 45 = 3^2 \times 5. \]

Then AF determines that the GCD of 45, 30, and 15 is \( 3 \times 5 = 15 \). Finally, AF wrote that 15 days after April 30, namely May 15, 2020, they would buy credit together.

To obtain in-depth information regarding the students’ process in solving problem number 1, the researchers conducted an interview with AF student. The results of AF student interview on problem number 2 are presented in table 2 below.

Table 2 Analysis of AF student interviews in solving problem number 2

<table>
<thead>
<tr>
<th>Reading:</th>
<th>The student correctly read the question</th>
</tr>
</thead>
<tbody>
<tr>
<td>Please read the question to me.</td>
<td>Looking for the date they bought credit together again”</td>
</tr>
<tr>
<td>Comprehension:</td>
<td></td>
</tr>
<tr>
<td>Tell me, what is the question asking you to do?</td>
<td>I am looking for the GCD of 15, 30 and 45.</td>
</tr>
<tr>
<td>Transformation:</td>
<td></td>
</tr>
<tr>
<td>Tell me how you are going to find the answer</td>
<td>Students show the result of his work, as shown in Figure 1. First, divide all 15, 30, and 45 using a factor tree. Then determine the prime factorization of 15, 30, and 45. Next, I choose the most minor and the same</td>
</tr>
<tr>
<td>Process Skills:</td>
<td></td>
</tr>
<tr>
<td>Show me how you get your answer, and “talk aloud” as you do it, so that I can understand how you are thinking.</td>
<td></td>
</tr>
</tbody>
</table>
Analysis of MF's work

MF views question number 1 as GCD problem. MF does not explicitly state the information that is known and asked in the question. Next, MF makes a factor tree of the numbers 84, 60 and 96. Then, MF wrote the prime factorization, namely $84 = 2 \times 2 \times 3 \times 7 = 4 \times 3 \times 7 = 12 \times 7$; $60 = 2 \times 2 \times 3 \times 5$; $96 = 3 \times 2 \times 2 \times 2 \times 2 \times 2$. Figure 2 below is the result of MF's work on number 1.

Figure 2
MF's Work on Number 1

Figure 2 shows that there is an error in writing the factor tree of the number 96. MF divides the number 96 by the number 3. MF should first divide the number 96 with the smallest prime number, namely 2. This results in errors in determining the factorization of prime numbers from 96. To obtain in-depth information about the MF process in solving problem number 1, the researchers conducted interviews.

Based on the interview results, MF marked the keyword "amount" contained in the question as a sign that question number 1 is a question related to GCD. When the researcher asked MF to explain question number 1, MF stated that this question aimed to find GCD. The following are the results of interview analysis based on Newman's stages with MF on question number 1.
Table 3. Analysis of MF student interviews in solving question number 1

<table>
<thead>
<tr>
<th>Reading: Please read the question to me.</th>
<th>The student correctly read the question</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comprehension: Tell me, what is the question asking you to do?</td>
<td>The presence of the word &quot;amount&quot; in the question makes me think that this is about GCD.</td>
</tr>
<tr>
<td>Transformation: Tell me how you are going to find the answer</td>
<td>Create a factor tree, determine the GCD</td>
</tr>
<tr>
<td>Process Skills: Show me how you get your answer, and &quot;talk aloud&quot; as you do it, so that I can understand how you are thinking.</td>
<td>Student shows the results of his work as in Figure 2</td>
</tr>
<tr>
<td>Encoding Now, write down your actual answer.</td>
<td>Answer $2^2 \times 3 \times 5 \times 7 = 420$</td>
</tr>
</tbody>
</table>

For question number 2, MF considers that this problem is solved using the LCM method. Figure 3 below is the result of MF's work on question number 2.

Figure 3
MF's work on number 2

Figure shows that MF cannot explicitly state the known information and is asked in the question. Next, MF created a factor tree of the numbers 15, 30, and 45. Then, MF wrote the prime factorization, namely $15 = 3 \times 5$; $30 = 2 \times 3 \times 5$; $45 = 3 \times 3 \times 5$. MF determined that the LCM of 15, 30 and 45 is $3^2 \times 5 \times 2 = 810$. To obtain in-depth information regarding the results of MF's work on question number 2, the researchers conducted interview with MF.

Table 4. Analysis of MF student interviews in solving problem number 2

<table>
<thead>
<tr>
<th>Reading: Please read the question to me.</th>
<th>The student correctly read the question</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comprehension: Tell me, what is the question asking you to do?</td>
<td>The presence of the word &quot;amount&quot; in the question makes me think that this is about GCD.</td>
</tr>
<tr>
<td>Transformation: Tell me how you are going to find the answer</td>
<td>Create a factor tree, determine the GCD</td>
</tr>
<tr>
<td>Process Skills: Show me how you get your answer, and &quot;talk aloud&quot; as you do it, so that I can understand how you are thinking.</td>
<td>Student shows the results of his work as in Figure 2</td>
</tr>
<tr>
<td>Encoding Now, write down your actual answer.</td>
<td>Answer $2^2 \times 3 \times 5 \times 7 = 420$</td>
</tr>
</tbody>
</table>

Radit, Fikar, and Budi will take an online mathematics course. To support the implementation of online courses, they need credits. Radit buys credit every 15 days, Fikar buys credit every 30 days, and Budi buys credit every 45 days. If they
bought credit together on April 30, 2020, when will they buy credit together again?

Reading: Please read the question to me. The student correctly read the question.

Comprehension: Tell me, what is the question asking you to do? This question is about LCM.

Transformation: First, 15 30 45 a factor tree is created, then write prime factorization and determine the LCM.

Process Skills: After making the factor tree, I choose all the highest prime numbers as the LCM.

Encoding: Now, write down your actual answer. $LCM = 3^4 \times 5 \times 2 = 810$

RESULT AND DISCUSSION

This study reveals the errors experienced by students in solving the word problems of LCM and GCD. The results showed that there were students who answered correctly, incorrectly, and did not answer. Students who experience errors in solving the LCM and GCD word problems tend to experience confusion in distinguishing the LCM and GCD problems. The phenomenon experienced by the student is due to the similarities in the topics of the LCM and GCD. The similarity occurs when choosing prime numbers in determining the value of the LCM and the value of GCD. This phenomenon is often known as interference. (Slavin, 2018) states that interference occurs when remembering information in long-term memory is used to solve problems in working memory, or short-term memory is exchanged with other information.

Another finding that is also a concern in this study is that students cannot solve the problems of the LCM and GCD. Students who do not solve the LCM and GCD word problems tend not to have a productive disposition, which is a belief in their ability to solve mathematical problems (Woodward et al., 2017). However, the main focus in the discussion of this research is the description of the students' errors in solving the LCM and GCD word problems. In addition to reviewing the results of student work, an interview process based on Newman's guidelines was carried out to obtain in-depth information regarding student errors. The types of errors experienced by students in this study included comprehension errors, transformation errors, processing skills errors, and encoding errors.

Comprehension Errors

The interview results showed that AF could read the problem well. However, the AF could not understand problem number 2. AF students looked at problem number 2, which was solved by the GCD method. It is the results in errors at the stage of comprehension of the problem. This finding aligns with the findings (Clements, 1980; Singh et al., 2010), stating that students who experience errors in the understanding stage will ignore important information to impact students' plans to solve problems. (Reid O'Connor & Norton, 2020) also stated that students' errors at the comprehension stage were also caused by the inability of students to connect relevant information to problem-solving solutions.
Transformation Errors

The results of the work and interview analysis of AF student also shows errors in the transformation stage. AF student experienced an error in determining how to solve the LCM problem. According to AF, problem number 2 was solved utilizing GCD. It is in line with the findings (Clements, 1980), which states that transformation errors can occur if students cannot change written problems in the form of correct mathematical procedures. It also revealed that transformation errors occur when students fail to identify mathematical procedures appropriate to the problem (Sukoriyanto & Desmayanti, 2021).

Process Skills Errors

Process skill errors occur when students experience calculation errors (Clements, 1980; Reid O’Connor & Norton, 2020; Wijaya, 2014). This study also shows that there are student errors in the process skills stage. Process skill errors experienced by MF were errors in making a factor tree of 96. In addition, other errors occurred when MF chose prime numbers in determining the value of the LCM and the value of GCD. In this study, errors occurred in MF students who chose all prime numbers with the highest order as the LCM value. (Reid O’Connor & Norton, 2020) in his study, students should perform mathematical calculations correctly at the skill stage of the problem-solving process to support completing problem-solving tasks.

Encoding Errors

Errors in the encoding stage occur when students fail to write down the appropriate conclusion (Clements, 1980; Reid O’Connor & Norton, 2020). In this study, examples of encoding errors were experienced by AF and MF students. Both did not write the conclusion of the answer correctly. It is also seen through the results of interviews with AF and MF students. Student errors at the encoding stage result from student errors in comprehension, transformation, and process skills.

CONCLUSION AND IMPLICATION

Conclusion

The productive disposition of students influences students who do not answer questions number 1 and number 2. Students who do not answer questions tend to think that they are unable to solve problems. In addition to students who answered correctly and did not answer the problem, this study also examined examples of errors experienced by students in solving problems. Most students experienced errors at the stages of comprehension, transformation, processing, and encoding skills.

Examples of students who experience comprehension errors are students who can read the questions well but do not understand the direction of solving the LCM or GCD problems. Students who experience transformation errors, namely when students determine the method used in solving problems, for example, students solve the LCM problem using the GCD method. Errors in the process skills stage occur when students experience errors in writing the factor tree of a number. Students also experienced errors determining the value of the LCM and the value of the GCD because there was an exchange in the principle of choosing prime numbers. Students determine the value of the
LCM by choosing the prime number with the minor power and determine the value of the GCD by choosing the prime number with the highest order. Errors at the encoding stage occur when students do not write down the sweeping conclusions of the LCM and GCD problems.

Implication

An implication of this is the possibility that the next researchers can conduct the study related to the student's disposition in solving a mathematical problem.

REFERENCES


