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Inductive reasoning skills: Identification of student errors using APOS theory

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a b s t r a c t

Math problem-solving mistakes made by students need to be treated more seriously since, if left unattended, they may affect students' comprehension of the following mathematical ideas. This study aims to identify students' errors while attempting to solve mathematical problems of inductive reasoning ability on the material of powers and roots. This study adopts a qualitative method with descriptive research. The participants in this study were 24 students of class IX A at SMPN 2 Pelalawan. The research methods used are tests, interviews, and triangulation. The results showed that students had low inductive reasoning abilities so they made mistakes in solving problems. Based on the APOS theory (action, process, object, and schema), there are four student errors in solving math problems, namely: 1) incorrect interpretation: students do not grasp the meaning and objective of the questions; 2) conceptual error: misunderstood the multiplication of powers; 3) procedural errors: they occur when students conclude problems without adhering to a process; 4) Technical errors: Students frequently make errors while multiplying powers because they do not understand the concept. Regarding the APOS theory, based on the data analysis students only arrived at the first phase, namely the action phase.

K e y w o r d s :
Apos theory, Inductive reasoning, Identification errors

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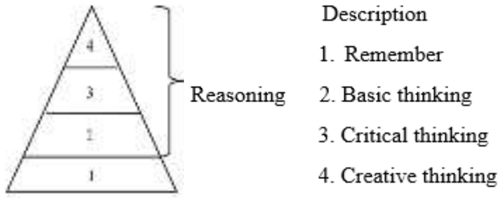
INTRODUCTION

Mathematical mistakes made by students need to be handled more seriously because, if they are not, they might affect students' comprehension of or knowledge of other mathematical ideas, such as the material on powers and root forms connected to facts, concepts, principles, and other abilities. According to (Siyepu, 2015) Identifying mistakes might let math teachers concentrate on creating instructional strategies that could help students with difficulties. Meanwhile, according to (Kazunga & Bansilal, 2020) Calculation mistakes in manipulating numbers and conceptual errors are common causes of errors. Therefore, reasoning and knowledge required to support student's answers (Corcoran & Frith, 2005; Kantahan et al., 2020).

The field of math is among the sciences in the realm of education (Barton, 1996). Math is taught at all educational levels, from elementary school through college (Ding et al., 2006; Seah et al., 2021). Mathematics is a branch of science frequently utilized to address numerous issues in various fields (Andamon & Tan, 2018; Retnawati et al., 2018; Weinhandl et al., 2020; Yadav, 2019). According to (Khan, 2015) mathematics is not just about solving everyday problems; it also involves applying creativity, intuition, and logic to generate original concepts and unravel complex issues. Meanwhile, according to (Widiati & Juandi, 2019) The key to other branches of science is mathematics. Students must be able to reason, which is a crucial skill in learning mathematics (Junpeng et al., 2020; Sidenvall et al., 2015; Sosa-Moguel & Aparicio-Landa, 2021). This is cited in Permendiknas No.22 of 2006 (2006) that the goal of mathematics education is for students to comprehend mathematical ideas, reasoning to patterns and attributes, and solve problems, including the ability to comprehend difficulties. For this reason, teachers must also possess various competencies to help students understand (Bakar, 2018). Comprehensive understanding of mathematical concepts, one of which can be demonstrated through reasoning (Quilang & Lazaro, 2022).

Making inferences is a unique type of mental action called reasoning (Agustyaningrum et al., 2019; Yanto et al., 2019), Through logical reasoning, learners gather pertinent data, interpret it based on their interpretation, and then develop several assertions that help them justify and draw conclusions (Buetow, 2014; Mehadi, 2019; Muhammad & Yolanda, 2022). Meanwhile, according to (Ponticell, 2019) A subset of higher order thinking, which includes mental processes like comprehending, analyzing, and making decisions, is reasoning. According to (Mercier et al., 2017) The mechanism of reasoning deals with the connection between reasons and conclusions. Even though it is not engaged in the overwhelming majority of cognitive functions, reasoning in this sense would be essential to critical thinking (Palinussa et al., 2021). Therefore, based on some of the perspectives of specialists, it may be inferred that reasoning is a mental activity in which a person formulates ideas, makes choices and draws conclusions based on the available data. The stages of reasoning thought are basic, critical, and creative thinking, the degrees of reasoning are listed below



Source : (Zuhri & Purwosetiyono, 2019)

Figure 1.

Levels Of Reasoning

From the picture above, it can be seen that a higher level of thinking than remembering is reasoning. The phase of reasoning begins with basic thinking and go up to a more advanced level, which is critical thinking. At this phase, One of the skills students need in the twenty-first century is critical thinking. Then move up to the last level, which is creative thinking. Deductive and inductive reasoning are two main categories into which reasoning is often classified (Davis, 2018; Kim et al., 2021; Mastropasqua et al., 2010). According to (Khansa et al., 2020; Mukumbang et al., 2021) Deductive reasoning involves drawing inferences from the general to the particular, while inductive reasoning involves creating a new general statement from specific examples. Meanwhile, according to (Bhat, 2016; Soysal & Soysal, 2022) On the basis of specific facts, one might formulate generalized rules and judgments using inductive reasoning. However, deductive reasoning is a method of obtaining conclusions from evidence and logic. With this kind of reasoning, one may make logical deductions from known reports or signs. So, Inductive reasoning involves creating a generalization from a particular instance, while deductive reasoning involves drawing inferences from the general to the specific.

The APOS (Action, Process, Object, Schema) theory is one of several approaches that may be used to investigate students' thought processes in mathematical reasoning (Umam & Susandi, 2022). According to (Arnon et al., 2014) APOS theory is a constructivist theory concerned with how a mathematical concept is studied. According to (Salgado & Trigueros, 2015) APOS theory has four phases, which are as follows:: 1) Action stage, students engage in procedural tasks using mathematical ideas that have action steps; 2) Process stage, with explanations and words, students can explain the stages of work, starting from the action stage, so they can understand the procedure; 3) Object stage, Students may provide examples of the subject of powers and forms of roots, but they can also explain what something means or how to do it; 4) Schema stage, students may make connections between the action stage, the object, and the problem-solving process in order to answer the provided issue.

Table 1.

Percentage of KKM achievement of class IXA students of SMPN 2 Pelalawan

Subject Matter	KKM	Number Of Students	Achievement of KKM	
			Amount	%
Powers and roots	76	24	6	25%
Quadratic Equation	76	24	10	41,6%
transformation	76	24	7	29,1%
Average				31,9%

Data Source: Math Teacher at SMPN 2 Pelalawan

Following the findings of an interview with a teacher, students often make calculation errors while completing math problems, particularly when it comes to powers and roots, and only six out of 24 students in class IX at SMPN 2 Pelalawan received exam scores that were above KKM (76). This demonstrates high student error rates, resulting in low learning results, as shown by table 1 about the achievement of students' KKM on certain materials. Based on the results of research related to inductive reasoning and student errors, that is from the results of research (Wu & Molnár, 2018) concluded that problem-solving training programs should be supplemented by instruction in specific reasoning abilities since inductive reasoning has a substantial predictive influence on problem-solving and affects accomplishment. Meanwhile, Arnon concluded about student faults using the APOS theory, where Arnon advised that further study be done to identify students' problem-solving thought processes. Finding out how kids think can help us better understand why students pick particular paths over others; teachers who understand how kids think can better identify whether a student is having trouble with a math issue (Arnon et al., 2014).

This research aimed to identify students' mistakes while using inductive reasoning to solve power and root form issues using the APOS theory. The author chose the APOS theory because it can explain how students' mental processes help them construct mathematical notions.

Methods

This study adopts a qualitative method with descriptive research. Qualitative descriptive research is a study that uses observation, interviews, and documentation to describe phenomena or event that happens in the field and delivers factual, accurate, and systematic data (Atmowardoyo, 2018). To verify the accuracy of the data, researchers employ triangulation.

Participants in this research are all Class IX A SMPN 2 Pelalawan students. Because not all students were chosen to participate, participants were chosen using a purposive sample approach. The number of students in class IX A is 24 students. All 24 students have studied the material of powers and roots. Because the 24 students have studied the material strength and roots. Therefore, then the 24 students will do the test. Students completed an 80-minute test with five questions that explore their capacities of inductive reasoning ability, and once the allotted time had passed, they turned in their response papers. Based on signs of inductive reasoning, all response sheets are examined. Participants who replied correctly or exceptionally well were not chosen as participants; instead, students who answered incorrectly or did not perform well enough when answering questions and solving problems involving powers and root forms were chosen as participants. In order to conduct interview activities, participants must be able to communicate.

Research instrument

Tests, and interviews were the research instrument employed. The main purpose of the test is to find various mistakes made by students. The test contains five questions that can be used to assess students' mathematical inductive reasoning abilities, the entire test has met the indicators of inductive reasoning (Atmowardoyo, 2018), namely: 1) demonstrating mathematical statements both orally and in writing; 2) presenting conjectures; 3) mathematical manipulation; 4) determining the validity of an argument; 5) drawing inferences and offering justifications or evidence; 6) identifying patterns that point in the direction of generalization; and 7) drawing conclusions from the statements that are already in existence.

Students' errors in answering questions may be identified from the outcomes of their test-taking responses. However, the causes of these errors and other relevant information can also be discovered by conducting interviews with students who have made similar errors. Because the questions are based on student response sheets, the interview style is unstructured, or the questions are not scheduled. Students must be able to respond and explain their reasoning for the outcomes of their replies during the interview activity, which is conducted in a lighthearted manner.

Data analysis

Initially, 24 students were assigned to take a test consisting of five essay-style questions about the ability of inductive reasoning. They are given 80 minutes to complete the test and submit the answer sheet after time runs out. In the next step, the researcher chooses one of the students' answers based on assumptions and mistakes made by students. This is discussed further in the discussion section of this study. After participants the research was selected, and interviews were held to confirm additional responses to students' written responses.. The interview, which lasted around 30 to 45 minutes, was the next phase in which the researcher used the student response sheets that had been checked to conduct interviews. Interviews were conducted when the research was taken four students based on the number of mistakes made by students when solving problems powers and root forms. The APOS theory, which seeks to discover or gather information linked to the errors committed, was applied in the recording and transcribing of interviews. The questions are based on the results of the student's answer sheets, whether they understand the meaning of questions one to five, what are their steps in working on the questions, and what they understand about the material for powers and root forms. Following that, each researcher individually examined each student's work. The data from the interviews, the student's responses, and the study outcomes were then triangulated. After analyzing the outcomes of the data triangulation, the researcher may conclude

Findings and Discussion

Finding

The test results of students who err on the first question, which includes the first indication (demonstrating mathematical statements both orally and in writing), and the second indicator, are shown in the image below (presenting conjectures). Whereas in the second indicator, students make deviations in presenting allegations that lead to errors, the error lies in the number three, which should be filled with five to the power of three.

Interviews are conducted using the APOS theory to identify student errors and develop inductive reasoning skills in math case material powers and root forms. Interviews were conducted based on APOS theory, questions at the stage of action, process, object, and scheme, namely whether students were able to explain what they had written (action), were able to explain the stages of work from the action stage with explanations and words so that they had understanding in the process (process), students are able to explain the meaning or definition and provide examples and not examples on the material of powers and root forms (objects), and students can link between the three previous stages, namely the action stage, object, process used to solve the problem in solving the given problem (scheme).

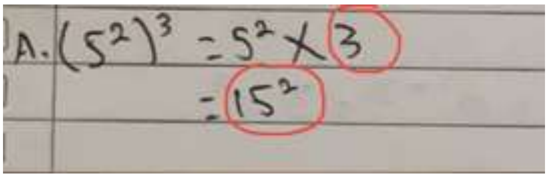


Figure 2.
First Question

The interview findings revealed that students' lack of understanding of multiplying powers caused them to present incorrect conjectures. After learning to memorize information, students quickly forget what they had previously learned.

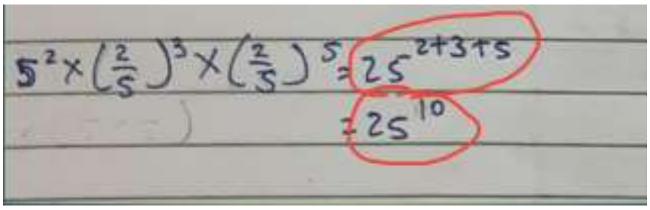


Figure 3.
Second Question

Students who erred on the second, which includes the third indicator (mathematical manipulation) had their test results in the image above. Students make errors in mathematical manipulation in this second question, which affects how they respond to questions. the error lies in the number 25 which should still be written 5 to the power of 2, then multiplied by 2 divided by 5 to the power of eight. In order to discover student faults in responding to questions, interviews were done based on the APOS theory since this indicates that students have not shown the markers of inductive reasoning skills. The interview's findings revealed that students struggled with fractional problem-solving, which indicated that they erred technically in their division calculations. It may be assumed that students do not possess a solid understanding of the necessary material, such as fraction operations.

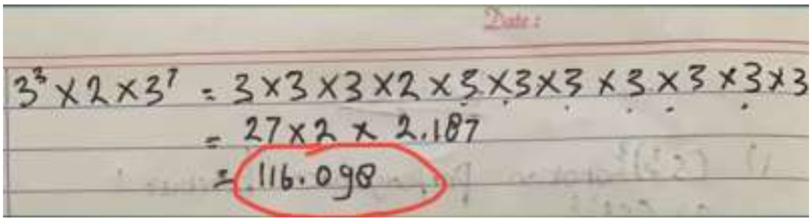


Figure 4.
Third Question

Students' test scores for the third question, which includes the fourth indicator (determining the validity of an argument) are shown in the image above. From the results of the student answer sheets, this question is where most of the students make mistakes. Students err in their assessment of the veracity of an argument in this third question, which affects how they respond to questions. This shows that the students have not reached the markers of inductive reasoning abilities (Atmowardoyo, 2018). Hence interviews using the APOS theory were done to determine the mistakes students made while responding to questions. The findings from the interviews showed that students made technical mistakes in multiplication operations, which led to inaccurate final results. This was due to students' failure to double-check their work and their sloppier calculation techniques. Students lack proficiency in multiplication operations involving three or more numbers and do not grasp the idea of multiplication, according to tests of student proficiency in the subject.

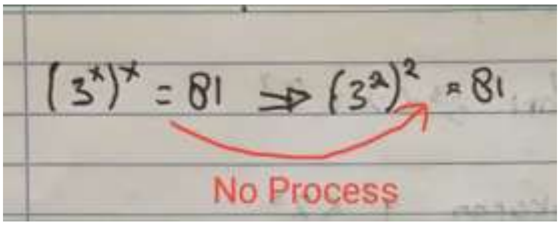


Figure 5.
Fourth Question

Drawing conclusions from existing statements is the seventh sign in the image above, which also includes the fifth indicator, forming inferences. According to the findings of the response sheets, students merely write answers without outlining their procedure or processes for completing the questions, and they also fail to draw inferences from the results, which indicates that they interpreted the material incorrectly. students directly answer questions or immediately conclude that the value of x is equal to 2. Whereas in drawing a conclusion certain facts are needed (Bhat, 2016; Soysal & Soysal, 2022). students should make steps to solve the problem. The students did not understand the purpose of question number four, and they did not take action to solve the problem. As a result, it can be assumed that students made procedural errors due to the lack of completeness in answering the question. An interview was conducted based on the APOS theory to identify errors.

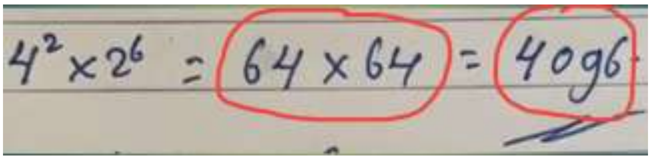


Figure 6.
Fifth Question

From the picture above which contains the sixth indicator, namely getting a pattern in the direction of generalization contained in the last question, where students make procedural errors, the student's error lies in the number 64 multiplied by 64 the student should simplify his power, not determine the result, to ensure these errors, interviews are carried out based on the APOS theory. APOS theory-based interviews were conducted to ascertain this, whether the student understood the question or not. Based on the results of the interviews, students do not understand the questions, so they make mistakes in interpreting where the question command simplifies power instead of determining the value of exponents. Then students also make procedural errors, as shown in the picture above, which contains the sixth indicator. Therefore, it may be concluded that students' inductive reasoning skills have not reached the indications.

Discussion

Students are less able to build their problem-solving skills and use methods for problem-solving because they have weak reasoning indications and low reasoning abilities (Wahyuni et al., 2019). The most frequent errors made by students while using inductive reasoning skills may be broken down into four categories: interpretive errors, conceptual errors, procedural errors, and technical errors. Interpreting errors in solving problems occur when the question command is asked to simplify the power, while students solve it another way or look for the value of the power. It means, students have not been able to interpret or do not understand the purpose of the questions given.

The second mistake is a conceptual error in problem-solving. Conceptual errors frequently happen when students are given multiplication power questions and add the powers without understanding whether or not the number is of the same number. For instance, when 2 to the power of 3 is multiplied by 4 to the power of 5, students often add their ranks, leading to problem-solving mistakes. It means, students do not understand the concept of multiplying squares and root forms. According to (Banamtuan et al., 2022) students often make conceptual mistakes when they cannot comprehend symbols and values due to carelessness while answering questions. Meanwhile, according to (Winarso & Toheri, 2021) students make mistakes on mathematics tests, due to conceptual errors that result from misconceptions of existing ideas. The most common mistake is a misunderstanding of the idea. (Agustyaningrum et al., 2018).

Procedural errors can be found in students' answers. What often happens is that there are no complete steps to get to the answers. Another procedural error is when students answer correctly or conclude correctly, but are wrong in carrying out multiplication, division, addition, and subtraction operations in the form of exponents and roots.

According to (Agustyaningrum et al., 2018) Procedural errors occur when students use steps or ways that are not ordered and systematic in solving problems.

When students perform calculations like addition, multiplication, division, and calculating the power of the root form, technical errors can be discovered. These technical errors can result in incorrect answers to questions, for instance, when students follow the procedure exactly but the results or conclusions are incorrect because the calculation was incorrect. Students need to acquire a conceptual grasp of mathematical computations as a result. Students with weak inductive reasoning skills are the participants who have been chosen. Some students can correctly answer each question on the exam, which consists of five questions and seven signs of inductive reasoning, while others make some errors. Students seldom study problems with this reasoning inquiry, which contributes to their limited reasoning abilities on the content of powers and root forms. Students are only used to working on inductive reasoning tasks that lack indications. Therefore, students are recommended to often practice with questions that help them hone their inductive reasoning skills.

This inductive reasoning ability needs to be developed. Therefore, an appropriate learning model is required while engaging in learning activities, including power and root form material. This is consistent with what was said (Pahrudin et al., 2020) the learning model employed by educators is one of the elements that contribute to student's mathematical thinking not improving. According to (Muin et al., 2018; Wasiran & Andinasari, 2019) The study's findings demonstrating that students' inductive reasoning abilities taught through creative problem solving are greater than traditional learning prove that inductive reasoning abilities may be acquired by learning creative problem-solving models. Meanwhile, from the research results (Lommatsch & Packenham, 2019; Stephens et al., 2020) Demonstrates that increasing inductive reasoning skills by training or studying logic is successful. Meanwhile, according to (Lestari & Jailani, 2018) Students' ability to reason can be enhanced by using metacognitive techniques.

APOS theory comprises four phases: action, process, object, and schema; however, based on data analysis, students only reach the first step, which is the action phase. The next phase, which involves stances, objects, and schemas, is challenging for kids because they lack mental structure (Umam & Susandi, 2022). based on the APOS theory, students struggle with the idea of powers and root forms, particularly when multiplying and dividing powers. Students also struggle with understanding the purpose of the given question, leading to errors in the completion process and in drawing conclusions. Students' lack of comprehension stems from their inability to properly comprehend the information necessary for integer operations and fractional operations, which is the required knowledge for multiplication and root forms. Additionally, students struggle to complete tasks using critical thinking or inductive reasoning skills. The instructors or teachers should provide students more time and training in the future to help them comprehend the idea of powers and root forms as well as the technique after taking into account the errors that students made while completing the power and root form problems. This is according to what was conveyed by (Agustyaningrum et al., 2018) the dominant error made by students is a conceptual error. In order to decrease mistakes in the solution of power issues and root forms, students must participate in problem-solving and be given the necessary space.

Knowing the causes of errors and learning more about students' thinking and understanding requires identifying and analyzing the mistakes students make when solving issues. It is advised that students prepare mentally well, grasp the prerequisite subject, and learn how to deal with faults that may occur. Teachers need to teach their students how to interpret concepts. Because mastering mathematics requires a strong conceptual understanding, students must comprehend the concept of multiplication of exponents and root forms to multiply powers correctly. This error identification is anticipated to assist current and future educators in identifying student errors, making it simpler to organize instructional activities. Errors made by students are due to low inductive reasoning abilities. According to (Wahyuni et al., 2019) The development of and difficulty in solving issues will be greater for those with poor reasoning skills. So, Students make four mistakes related to the APOS theory: 1) incorrect interpretation; 2) Misinterpretation of the concept; 3) procedural mistakes; 4) Technical errors.

Therefore, it is essential to strengthening students' inductive reasoning skills in order to decrease such mistakes. Students will not struggle to answer mathematics issues if their inductive reasoning skills have sufficiently developed or reached a certain level, making it simpler for them to do so.

Conclusion

Based on the study findings and discussion, students who participate have weak inductive reasoning skills, as shown by their failure to satisfy 7 inductive reasoning indications. Students' errors in addressing issues using mathematical inductive reasoning are the root cause of this incapacity. Students make four mistakes related to the APOS theory: 1) incorrect interpretation: students do not grasp the meaning and objective of the questions; 2) Misinterpretation of the concept of the multiplication of powers; 3) procedural mistakes: they occur when students conclude problems without adhering to a process or set of procedures or when they use unorganized, haphazard methods; 4) Technical errors: Students frequently make errors while multiplying powers because they do not understand the concept and are inaccurate when they answer problems.

This research is expected to help prospective teachers in choosing the right strategies, models, and learning media, especially in terms of rank and root form. The suggestions from the researchers from the results of this study are: 1) for teachers, before carrying out learning, the teacher must ensure students' understanding of the prerequisite material to be taught; 2) for teachers to always provide space for students to express their opinions and they can independently solve problems or in problem-solving to reduce errors in solving problems of rank and root form; 3) for further research to identify errors in other materials or follow up on mistakes made by students in answering math problems, and also for further research in order to be able to use other learning media, methods and learning strategies.

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