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Students' Arguments in Solving Probability Theory Problems Based on The Toulmin Argumentation Model

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a b s t r a c t

The role of arguments in solving mathematical problems is very important. Students must use valid arguments and concepts that have been learned in the Probability Theory material to build their arguments. An argument can be analyzed using the Toulmin scheme. Therefore, the purpose of this study is to describe the arguments that have been built based on the Toulmin Argumentation Model. The instrument used to collect data is a test question related to the Probability Theory material. Of the 32 students who worked on the test questions, three students were selected as research subjects. The selection of research subjects is based on the ability of students' mathematical ability level in working on test questions. It was found that students can construct an argument starting with the correct data. However, they did not give any Warrant and did not even give a Claim to their argument. This makes the argument built by students is invalid.

K e y w o r d s :
Argument; Probability; Students, Toulmin

INTRODUCTION

Mathematics learning in higher education needs to overhaul the mathematical knowledge acquired in school to enable examining the foundations of, and connections between, higher education mathematics and relevant mathematical applications (Singh, 2009). The quality of students' mathematical knowledge has always been crucial. An important factor that determines the quality of knowledge is the quality of students' experience in constructing their knowledge to solve a problem (Singh et al., 2016).

Problem-solving requires supporting arguments (Tristanti & Nusantara, 2021b). Stylianides (2007) defines an argument as a series of connected statements intended to verify or refute mathematical claims, or arguments can also be called the result of the reasoning process (Mercier & Sperber, 2011; Soekisno, 2015). The ability to argue is very important to define, express, and support reasonable solutions, provide descriptions to support or reject a premise, point of view, or idea, and not raise doubts about solving a problem (Tristanti & Nusantara, 2021a).

Until now, the level of students' argumentation skills is still relatively low, which can be seen from the low desire of students to respond to the lessons that have been taught, and responses to answers given by others (Pugalee, 2001). Some other studies show similar things. One of them is Singh & White's research which states that students are unable to unpack their mathematical knowledge and apply it to new situations (Singh & White, 2006). Then research by Singh et al. (2017) showed that students who get high scores on exams still have difficulty solving non-routine problems. This is because students only follow the directions given by the teacher in solving a problem. When faced with problems that have higher difficulty, students need to connect existing information using reasoning to solve the problems given (*PISA 2015 Results (Volume I)*, 2016).

Stephen Toulmin stated to analyze an argumentation can use the Toulmin scheme (Toulmin, 2003). Banegas (2013) states that the Toulmin scheme can be used in the analysis, assessment, and construction of arguments, and by using the scheme it can be seen whether the argument is supported by valid data, what guarantees are used to declare the argument valid, whether there is a refutation of the argument. Rosen (2011) states that an argument is valid if its form is valid, that is, if each premise is substituted with a certain statement, the results of all premises are true, and thus the conclusion is also true.

Toulmin's argumentation model (Toulmin, 2003) consists of three main components: Data (D), Claim (C), and Warrant (W), and three (3) complementary components: Backing (B), Rebuttal (R), and Qualifier (Q). Toulmin's argument is depicted in a schematic as in Figure 1.

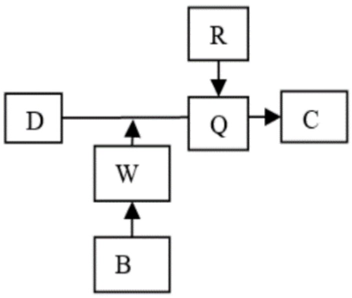

Figure 1
Toulmin's Argumentation Model

The benefit of analyzing an argument with Toulmin's Argumentation Model is to capture the best meaning or strength of words and propositions by seeing how one can use them in various contexts (Bizup, 2009). Example "If is n an integer then n^2 is an even number". The argument is true in the context of even numbers. However, it will be false in the context of odd numbers (Tristanti & Nusantara, 2021a). One of the main misconceptions that have developed about Toulmin's model is that it assumes arguments have six components when they do not (Nussbaum, 2011). Toulmin (2003) states that some components of arguments may be left implicit; in fact, Warrant is usually left implicit unless further clarity is required.

Huang et al. (2021) state that problems in probability and statistics often require reading material. In addition, many abstract solutions and concepts involved in probability and statistics problems make it difficult for students to understand the methods, and concepts to solve the problems.

Research related to argumentation develops in different formats and constructions, namely some that focus on the process of making and some that focus on the results of the argument (Muhtadi et al., 2020). Research related to the process of making arguments focuses on the construction of arguments based on the argumentation scheme to be achieved, and the essential dialogical elements (Labinaz, 2014). Then research related to the results of the argument emphasizes the structure of the argument (Toulmin, 2003), justification of the argument (Bergqvist, 2005), and types of arguments (Liua et al., 2016). However, the research to be carried out will focus on the results of arguments, not on the process of making arguments.

Based on the description above, this research aims to describe students' argumentation in solving problems of probability theory. Through this research, it is hoped that it can provide an overview of argumentation skills so that in the learning process students are better trained in providing arguments in the material of probability theory.

METHODS

This type of research is descriptive qualitative to describe students' arguments in solving mathematical expectation problems on Probability Theory. The subjects of this research were 2021 mathematics department students who took the Probability Theory course at one of the universities in Malang. The research subjects were taken using purposive sampling technique which amounted to 3 subject. The selection of these three subjects was based on the students' level of mathematical ability (high ability level (S1), medium ability level (S2) and low ability level (S3)). Both of these are used to obtain argumentation patterns from students with high, medium, and low levels of mathematical ability based on their level of mathematical ability.

Hitung $P(\mu - 2\sigma < X < \mu + 2\sigma)$, dimana X mempunyai pdf

$$f(x) = \begin{cases} 6x(1-x), & 0 < x < 1 \\ 0, x \text{ lainnya} \end{cases}$$

dan bandingkan dengan hasil yang diberikan jika menggunakan Teorema Chebyshev.

Figure 2
The test questions in this research

The instrument used in data collection consists of 1 questions contained in Figure 2. Data collection in this study used description questions. The question was used as a comparison of the arguments used by students in solving the problems given. The data analysis technique carried out in this study is by analyzing each student's argumentation and grouping it into elements that are in accordance with the Toulmin Argumentation Model. The data that has been obtained is then grouped and written in the form of narrative text.

RESULT AND DISCUSSION

Based on the results of the analysis of student argumentation ability test data in solving the problem of Probability Theory, and followed by 32 students, 3 subjects were selected, namely S1 as a subject with high ability, S2 as a subject with medium ability, and S3 as a subject with low ability. The three subjects showed diverse argumentation patterns, but none of the students managed to answer the test perfectly.

Subject 1 answered the problem tested on the test starting with finding the expectation value, because the expectation of the random variable X is the same as the mean (μ), and then finding the standard deviation value, by first finding the variance value. The following results of Subject 1's work to find the mean value are in Figure 3 below.

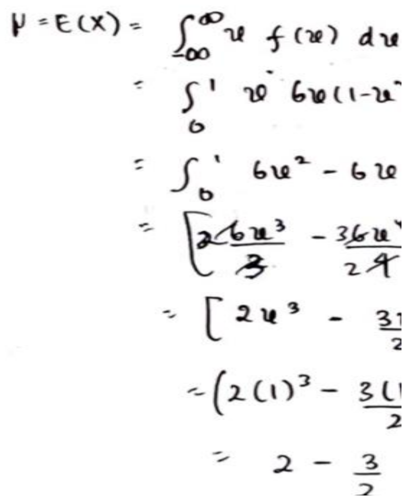

Figure 3
Subject 1's argument to get the mean value

Subject 1 used the mathematical expectation formula, probability density function ($f(x)$), and integral limits correctly according to the information in the problem, so Subject 1 used the correct data to build his argument. Then in the integral calculation process, Subject 1 was able to perform the calculation according to the integral concepts correctly without any errors. Therefore, Subject 1 was able to show that the Warrant used was valid to get the final result correctly. The claim submitted by Subject 1 is written implicitly, because several components of argumentation are presented implicitly (Toulmin, 2003), namely not directly mentioning that the mean value is 0.5, but based on the answer it can be concluded that the mean value is 0.5 which is correct. Because Subject 1 used Data, Warrant, and Claim which are true, it can be said that Subject 1 was able to provide a valid argument to determine the mean of the random variable X (Rosen, 2011).

Then Subject 1 calculated the variance (σ^2) to get the standard deviation (σ). The following is the result of Subject 1's calculation to get the standard deviation value in Figure 4 below.

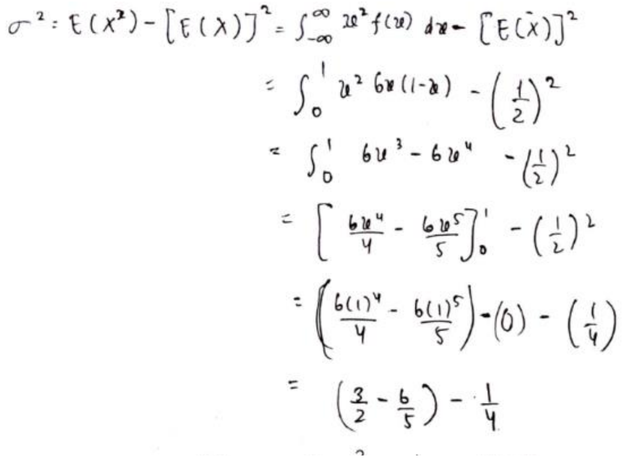

Figure 4
Subject 1's argument for calculating the standard deviation

Subject 1 used the Claim obtained in the previous argumentation, namely $\mu = 0.5$, as data to be substituted into the variance formula, namely $\sigma^2 = E(X^2) - \mu$. Then in the integral calculation process, Subject 1 could perform the calculation according to the integral concepts correctly without any errors, so Subject 1 used a valid Warrant to get a valid Claim. Similar to before, Subject 1 made the Claim implicitly, it was not directly stated that the variance value was $\frac{1}{20}$. However, based on the calculation results, it can be concluded that the variance value is $\frac{1}{20}$. Therefore, in determining the variance, Subject 1 was able to provide valid argumentation.

Then Subject 1 made a claim that $\sigma = \sqrt{\frac{1}{20}}$. The claim was certainly by using data, namely $\sigma^2 = \frac{1}{20}$ obtained from the previous calculation. However, Subject 1 did not provide any warrant in his work. Even though mathematically, it is possible for σ to be negative, namely $\sigma = -\sqrt{\frac{1}{20}}$, Subject 1 did not provide the reason why he only took positive values instead of negative ones. Therefore, Subject 1 provided an invalid argument for standard deviation.

After Subject 1 obtained the mean (μ) and standard deviation (σ), then Subject 1 substituted these values into the probability formula asked in the question, namely $P(\mu - 2\sigma < X < \mu + 2\sigma)$. The following are the results of Subject 1's work in calculating the exact probability value in Figure 5 below.

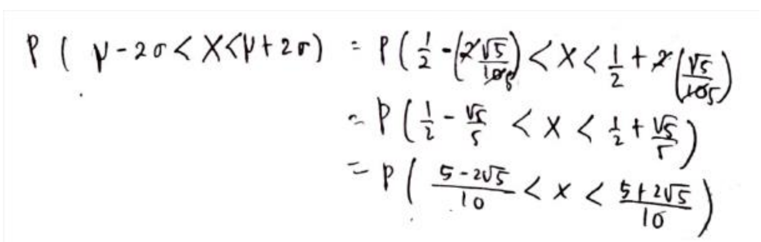

Figure 5
Subject 1's argument in calculating the exact probability value

Figure 5 above shows that Subject 1 was able to construct the probability formula correctly by connecting the previously constructed arguments used as data. However, in the next step, Subject 1 was unable to provide any Warrant and Claim in his argumentation. This is because Subject 1 did not have sufficient knowledge of the concept of probability of a random variable so he did not know how to calculate its probability value. Therefore, in calculating the probability value, Subject 1 failed to build a valid argument. Then the figure below is Subject 1's argumentation in determining the lower bound of the probability by using Chebyshev's Theorem

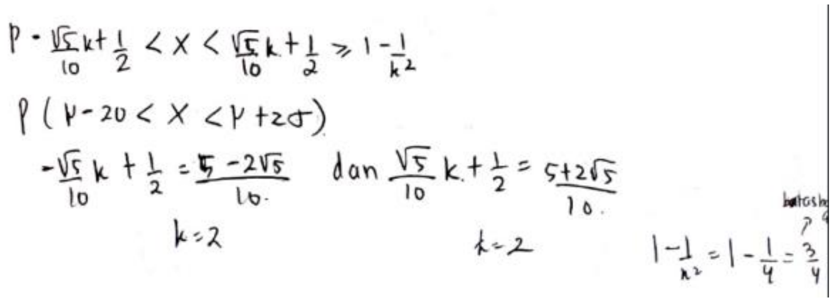

Figure 6
Subject 1's argument in using Chebyshev's Theorem

Based on Figure 6 above, Subject 1 was able to use the Claim obtained in the previous argumentation and combine it with Chebyshev's Theorem as Data. Although Subject 1 did not write down Chebyshev's Theorem, implicitly Subject 1 used Chebyshev's Theorem in the form below.

$$P(\mu - k\sigma < X < \mu + k\sigma) \geq 1 - \frac{1}{k^2}$$

To determine the lower limit, we must first determine the value of k, because the lower limit value is contained in the right segment. Subject 1 used the information contained in the problem, namely $P(\mu - 2\sigma < X < \mu + 2\sigma)$, and compared it with the lower limit of the probability formula in Chebyshev's Theorem. Then after comparing the resulting 2 equations, Subject 1 carried out the calculation process with the right mathematical concepts and produced the same k value for both equations, so Subject 1 was able to provide the right Warrant to produce the right Claim as well, namely the value of $k = 2$. The value of $k = 2$ then becomes the Data for the basis of reaching the Claim that the lower limit of the probability is $\frac{3}{4}$, with a Warrant in the form of a calculation process that is under the correct mathematical rules. Therefore, Subject 1 was able to provide a valid argument to determine the lower bound of the probability.

However, because Subject 1 failed to get the exact probability value, because it did not provide any Warrant or Claim, so Subject 1 was not able to compare the exact probability value with the lower bound of the probability obtained using Chebyshev's Theorem. Overall, the argumentation skills of Subject 1 were good enough by providing valid arguments, although some still failed.

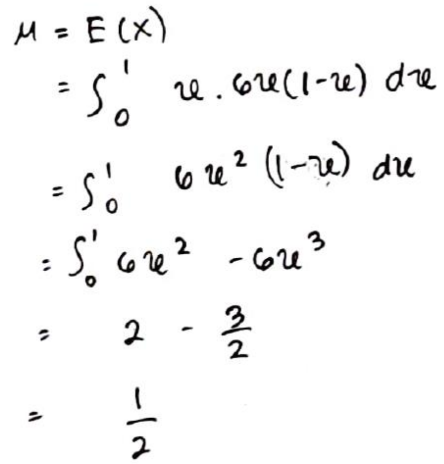

Figure 7
Subject 2's argument for calculating the mean

Based on Figure 7 above, Subject 2 started answering the test question by calculating the value of μ first. Subject 2 was able to use the mathematical expectation formula correctly and substitute $f(x)$ and the limit of the random variable correctly, so Subject 2 provided valid data used for calculating the integral value. However, Subject 2 did not provide any Warrant in his argument, there was no integral calculation process which is something important to get the value of µ. This makes the final result of the calculation process questionable. However, Subject 2 got the correct final result, namely, the mean value is 1/2. Based on the results of the work, Subject 2 was able to use the correct data and the correct Claim as well, but with no Warrant. Although the Claim is true, the argument built by Subject 2 is still invalid.

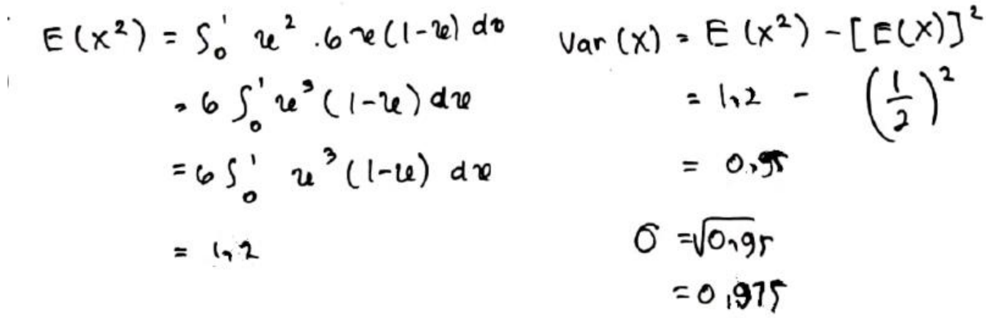

Figure 8
Subject 2's argument for calculating the standard deviation

Figure 8 above shows Subject 2's argument to find the standard deviation (σ). Starting with calculating the value of $E(X^2)$, Subject 2 used the mathematical expectation formula correctly and also used the information contained in the problem, namely $f(x)$ and the probability limit correctly, so that Subject 2 was able to use the correct data in calculating the expected value of X^2. However, as in the previous argument, Subject 2 was still unable to provide Warrant in his argument. It can be seen from Figure 8 above, Subject 2 did not describe the integral calculation process, even though the calculation process is an important part of determining the expected value. Subject 2 immediately gave the final answer, which is implicitly a Claim in his argument, namely the expected value is 1.2. However, the Claim made by Subject 2 is a false Claim because the expected value of X^2 should be 0.3.

Based on this, Subject 2 was only able to provide correct data in his argument, and was unable to provide the correct Warrant or Claim. This had an impact on the process of calculating variance and standard deviation. Subject 2 used the wrong Claim which was then used to calculate the variance and standard deviation values. As a result, the variance and standard deviation values obtained will also be wrong because Subject 1 used the wrong Claim in the previous argument, which was used as data to obtain the variance and standard deviation values. Then it also had an impact on the process of calculating the probability value by Subject 2 in Figure 9 below.

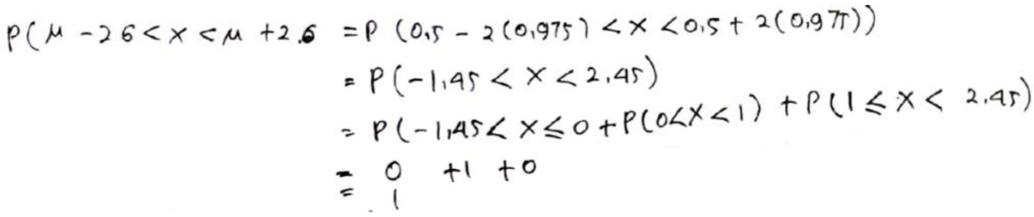

Figure 9
Subject 2's argument in calculating the exact probability value

Because Subject 2 used the wrong Claim in the previous argument, the value substituted in the calculation of the probability value, namely the value of σ, made the data used to be incorrect. Based on Figure 9, it can also be concluded that Subject 2 still did not understand the concept of probability correctly. This can be seen from Subject 2 translating $P(-1.45 < X < 2.45)$ into

$$P(-1.45 < X \leq 0) + P(0 < X < 1) + P(1 \leq X < 2.45)$$

Although the explanation can be said to be correct, it is not commonly used to calculate the probability value. Just such an explanation will complicate the calculation process. The right step is to do the integral with the limit obtained from the correct substitution of μ and σ values. However, Subject 2 immediately got the final result of 1 without any integral process. Subject 2 was unable to provide Warrant in his argument. Therefore, the Claim in the form of the exact probability value is 1 is incorrect because the argument departs from incorrect data and automatically produces an incorrect Claim as well. Based on this, Subject 2 was unable to provide a valid argument to determine the exact probability value.

Then Figure 10 below shows Subject 2's argument to determine the probability limit using Chebyshev's Theorem.

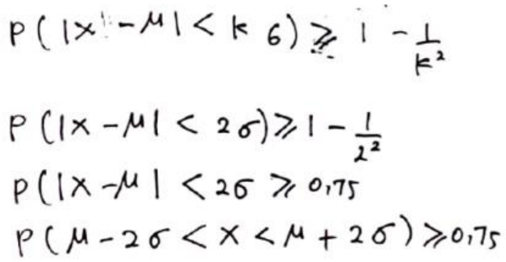

Figure 10
Subject 2's argument in using Chebyshev's Theorem

Subject 2 correctly wrote down Chebyshev's Theorem which was used as data. The next step should be to determine the value of k. However, Subject 2 did not write down how the steps to determine the value of k, but directly substituted the value of $k = 2$ into the Chebyshev Theorem, meaning that Subject 2 did not provide any warrant in his argument even though the value of k used was correct, and the lower bound obtained was also correct. However, the argument built is not included in a valid argument because it is without any Warrant. Because Warrant is an important component that must be present in an argument (Toulmin, 2003). Subject 2 also did not provide any argument in comparing the exact probability value with the results obtained by Chebyshev's Theorem.

The Figure 11 below shows Subject 3's argument for determining the mean value of the random variable X.

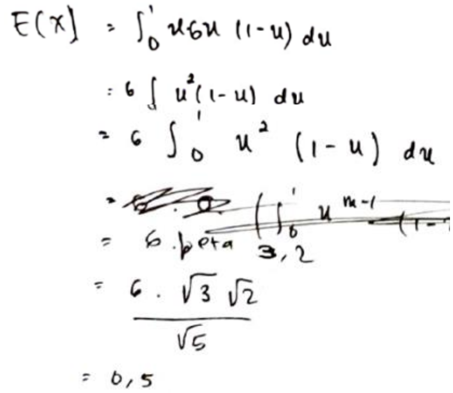

Figure 11
Subject 3's argument for calculating expected value

Similar to Subject 1 and Subject 2, Subject 3 was able to use data, namely information in the form of $f(x)$ and integral limits correctly. The next step is the integral calculation process. Subject 2 calculated the integral using the beta function, which is very unusual for calculating integral values unless the integral form is a special form. The beta function formula referred to by Subject 2 is as follows (From & Ratnasingam, 2022).

$$B(m,n) = \frac{\Gamma(m) \cdot \Gamma(n)}{\Gamma(m+n)} = \int_0^1 u^{m-1}(1-u)^{n-1} du$$

However, Subject 3 wrote $\Gamma(3)$ as $\sqrt{3}$, which shows Subject 3 still did not correctly understand the concept of the beta function related to the gamma function. Subject 3 did not explain the definition of the gamma function, which is $\Gamma(m) = (m-1)!$. Although the final answer (Claim) obtained was correct, namely 0.5, Subject 3 did not provide a Warrant that supported the Claim. Therefore, Subject 3 was not able to provide a valid argument in determining the mean value of random variable X. Furthermore, Subject 3 determined the expectation of X^2, then the variance to get the standard deviation value as in Figure 12 below.

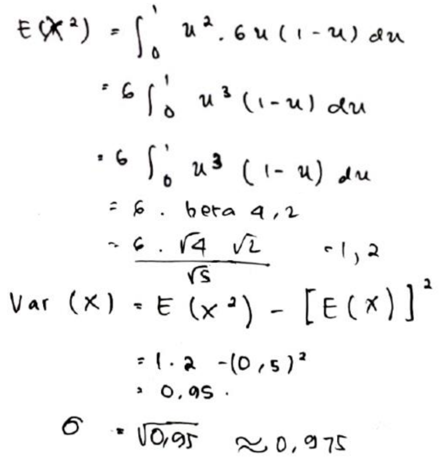

Figure 12
Subject 3's argument in calculating the standard deviation

Based on Figure 12 above, Subject 3 was able to use the data correctly just like the previous argument in determining the mean value. Subject 3 used the beta function to calculate the integral value and also still did not understand the beta function correctly. It can be seen from Subject 3 writing $\sqrt{4}$ when it should be $\Gamma(4)$ and without any warrant as a further calculation process. This had an impact on the wrong calculation results. So the claim that the value of $E(X^2)$ is 1.2 is not correct. As a result, the variance value and standard deviation value obtained are incorrect. Therefore, Subject 3 failed to build a valid argument to determine the standard deviation value. Similarly, for the exact probability value and the limit of the probability using Chebyshev's Theorem, Subject 3 did not provide any argument for this, which made Subject 3 a subject with low mathematical ability.

To see the difference in the correctness and validity of the argumentation constructed by subject 1, subject 2 and subject 3, Table 1 is presented below.

Table 1 Correctness and validity of arguments at each level of mathematical ability

Students' Ability	Components of Argument											
	Mean (μ)			Standard Deviation (σ)			Exact Probability Value			Chebyshev Theorem		
High	D	C	W	D	C	W	D	C	W	D	C	W
	✓	✓	✓	✓	✓	✓	✓	✗	✗	✓	✓	✗
	Valid			Valid			Not Valid			Not Valid		
Middle	D	C	W	D	C	W	D	C	W	D	C	W
	✓	✓	✓	✓	✗	✗	✗	✗	✗	✓	✓	✗
	Valid			Not Valid			Not Valid			Not Valid		
Low	D	C	W	D	C	W	D	C	W	D	C	W
	✓	✗	✗	✓	✗	✗	-	-	-	-	-	-
	Not Valid			Not Valid			Not Valid			Not Valid		

Note :
D : Data
C : Claim
W : Warrant
✓ : The argument component is true
✗ : The argument component is false
- : No answer
Valid : The argument is valid
Not Valid : The argument is not valid

Based on Table 1, it is found that subjects with high mathematical ability can use data correctly for all four argument components, able to make correct Claims in calculating the mean (μ), standard deviation (σ), and Chebyshev's Theorem, but failed to make correct Claims for the exact probability value due to the calculation of the integral which is quite complicated and at the time of the test was prohibited from using a calculator. Then subjects with high mathematical ability were able to provide correct Warrant in calculating the mean (μ), and standard deviation (σ), but failed to provide correct Warrant in calculating the exact probability value and Chebyshev's Theorem. Therefore, valid arguments that can be constructed by subjects with high mathematical ability are only found in calculating the mean and standard deviation. Subjects with moderate mathematical ability only failed to use the Data correctly in showing the exact probability value, because the Data used came from Claim in the calculation of standard deviation while he failed to give the correct Claim in the calculation of standard deviation. Then the subject with moderate mathematical ability was able to give the correct Claim in the calculation of the mean and Chebyshev's Theorem, but only gave the correct Warrant in the calculation of the mean, while in Chebyshev's Theorem, he did not. Therefore, subjects with moderate mathematics ability were only able to construct valid arguments in the calculation of the mean. Subjects with low mathematics ability were only able to use the correct Data in calculating the mean and standard deviation values, without providing the correct Claim and Warrant, nor did they provide any argument in calculating the exact probability value and the probability value using Chebyshev's Theorem. Therefore, all arguments constructed by subjects with low mathematics ability were invalid.

Based on the analysis of the answers of each subject, it is found that in constructing an argument, it must begin with correct data to support the correct claim as well. Incorrect data cannot be used as evidence to support claims (Aaidati et al., 2022). Because data is the starting point for building an argument (Banegas, 2013). Most students have been able to use the correct data, but are still constrained in providing the correct warrant. It can be seen that there are still many arguments that are built without using the correct Warrant or even without using any Warrant. This is in line with Nadlifah & Prabawanto's research which states that students often realize and can apply the facts needed to prove a statement but still fail to prove it by not providing Warrant for the statement (Nadlifah & Prabawanto, 2017). Correct data and Warrant are very important in an argument because a valid argument is based on Data, Warrant, and Claim that are true, and vice versa, if one of them is false then the argument built is invalid (Rosen, 2011).

The Warrant component of reasoning is one of the difficulties of students in building arguments due to a lack of understanding of the context of the problem, lack of mastery of mathematical concepts, and failure to connect between these concepts (Lizotte et al., 2003; Sholihah et al., 2021). Because the concepts in mathematics are closely related to the order between elements that are organized and arranged hierarchically, where the concepts in the previous material will be used in the next material (Hasratuddin, 2014). The role of the Warrant is very important as a guarantor that connects Data and Claims (Metaxas et al., 2016). Without the correct Warrant, the argument is not valid.

Based on the results of student answers, it is found that the components of the argument are not all presented explicitly, several components occur implicitly. This is following Toulmin (2003) who says that some argument components may be absent or left implicit. As in the case of students not writing the Claim to be proven, indirectly the Claim can be seen in the calculation process.

CONCLUSION AND IMPLICATION

The arguments built by students in solving Probability Theory problems using Data, Warrant, and Claim were found and most of them were still not valid. Subjects with high mathematical ability were able to construct arguments using correct Data and Claims, and supported by correct Warrants but failed to provide valid arguments to calculate the exact probability value. Subjects with moderate mathematical ability used correct Data and Claims but still failed to provide correct Warrants, especially in determining the lower bound of the probability using Chebyshev's Theorem. Subjects with low math ability were only able to provide correct Data but the Claim written was wrong and without Warrant.

In general, students can use the correct data but still fail to provide Warrant. This is due to students' lack of understanding of mathematical concepts and Probability Theory material which is a prerequisite in working on test questions. Because Warrant is the key to drawing conclusions based on existing data because Warrant is not appropriate, the arguments built will be invalid.

Based on the research results, a description of the arguments that have been built by students is obtained. However, it does not get a description of how the process of students building the argument. Therefore, it is necessary to conduct further research on the thought process in building an argument to know with certainty the student's argumentation ability.

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