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Comparison of Fuzzy Time Series Chen and Cheng to Forecast Indonesia Rice Productivity

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abstract

Based on BPS data in 2019, around 90% of Indonesians consume rice as the main product of their carbohydrate needs. With the fourth most populous country in the world, Indonesia's rice consumption in 2021 will reach 31.50 million tons. Indonesia's rice production is currently not able to supply the needs of domestic rice consumption. Therefore, the Indonesian government chose to import rice for supply the needs of domestic rice consumption. Forecasting method can help the government to reduce uncertainty about the future of rice productivity. This research will discuss the comparison of the forecasting results of fuzzy timeseries using chen and cheng models in forecasting rice productivity in Indonesia. The comparison based on the percentage of eror or in this research the MAPE value is used as an indicator. The MAPE value obtained using chen model is 18% and using cheng model is 12%. The result of data analysis found that the MAPE value using cheng model is smaller than using chen model. It means that the cheng model is more appropriate used in forecasting data of Indonesia productivity rice. However the chen and cheng models both give good forecasting result because the MAPE value is less than 20%.

Keywords:
Rice productivity, fuzzy time series, chen, cheng, MAPE

INTRODUCTION

Rice is the main staple food of Indonesia. Based on Badan Pusat Statistik (BPS) data in 2019, around 90% of Indonesians consume rice as the main product of their carbohydrate needs. The 2020 population census conducted by BPS for Indonesia population in September 2020 was 270,20 million people (Badan Pusat Statistik, 2021). This means that Indonesia must have a large enough supply of rice. But based on BPS data 2021, it shows that the harvested area and productivity of rice in Indonesia in 2021 has decreased compared to the previous year (Badan Pusat Statistik, 2021). There are many factors that affect the decreased of rice productivity such as rainfall, fertilizer quality, seed quality and human resources. Indonesia's rice production is currently not able to supply the needs of domestic rice consumption (Ishaq et al., 2017). Therefore, the Indonesian government chose to import rice for supply the needs of domestic rice consumption. However, rice import policies need to be carried out with caution. Rice import policy will have an impact on price stability if carried out excessively. Therefore, it is necessary to estimate domestic rice productivity so that the government can make appropriate rice import policies. Prediction of domestic rice productivity can be obtained using the forecasting method.

Forecasting is the art or science of predicting future (Heizer & Render, 2015). The purpose of forecasting is to reduce uncertainty about the future. The right forecasting method is needed to be able to provide accurate forecasting results so that it can be used as a reference to make the right decisions regarding future policies. Data on the movement of the value of Indonesian rice productivity follows a time series that is summarized annually. The time series forecasting method is carried out using previous data to determine future data (Hanke, et al, 2005). New approaches to forecasting techniques continue to be developed to obtain accurate predictions. Fuzzy logic is proven to have a better performance in solving uncertainty problems (Sofhya, 2020). Forecasting time series data using fuzzy models is known as fuzzy time series.

Fuzzy time series was first introduced by Song & Chissom (1993), The superiority of fuzzy time series is to defines a fuzzy relation that was built by determining the logical relation of the training data. Fuzzy relation is formed by training data logical relation which involves fuzzy sets from universal partition. Universal partition division based on statistical distribution on each partition. The use of statistical distributions as a consideration for repartitioning and the use of data for compiling universal sets is still an open problem in the context of how to determine the optimal forecasting model to improve forecasting performance (Maria et al., 2019). As science develops, there are several models of fuzzy time series was proposed to obtain optimal forecasting results. Some of them are models proposed by Chen (1996), Singh (2007), and Cheng (2008). These models have different ways of defuzzification to get the forecast value. Chen proposes using the average in obtaining the forecast value. Then, Singh proposes using a supremum in obtaining the forecast value while cheng uses the weighting in obtaining the forecast value.

The optimal forecasting model is determined based on the accuracy of the forecasting result. The mean absolute percentage error (MAPE) is one of the most popular measures of the forecast accuracy, it is recommended in the most textbooks (Kim & Kim, 2016). In some research, the forecasting methods are compared to obtain the best result determined based on the MAPE value. A forecasting method that produces a smaller MAPE value means a better method and more appropriate method to forecast the data. The research conducted by Arnita et al (2020) which compare three fuzzy time series model

to forecast rainfall in Medan shows that Chen model is the best model compared to the other two models based on the MAPE value. Then, the research conducted by Tauryawati & Irawan (2014) which compare Fuzzy time series cheng model with Box-Jenkins method to forecast IHSG, shows that from the two methods, the cheng model produces more accurate forecasting value with smaller MAPE values. Both studies show that the Chen and Cheng models are among the best methods for data forecasting. Therefore, this research will discuss the comparison of forecasting using fuzzy time series Chen and Cheng models in forecasting rice productivity in Indonesia. In this research, the comparison based on the MAPE value. The forecasting method that produces the smaller MAPE value means that those method is more appropriate method to forecast Indonesia rice productivity.

LITERATURE REVIEW

Fuzzy Set

The definition of fuzzy set was first introduced by Zadeh in 1965. Fuzzy set is a pair of (A, μ), where A is a set and μ is membership function μ: A→ $|0,1|$. For a finite set A=$\{x1, x2, \dots, xn\}$. Grade of membership of x in (A, μ) is $\mu(x)$. Let $x \in A$, x is not include of fuzzy set (A, μ) when $\mu(x) = 0$. In other ways x is fully included of fuzzy set (A, μ) when $\mu(x) = 1$. Then x is called member of fuzzy set (A, μ) when $0 < \mu(x) < 1$. When $\mu(x) > 0$, x is the support of (A, μ) and x is kernel when $\mu(x) = 0$. There are many ways to represent fuzzy sets, fuzzy sets (A, μ) can denoted by $\left\{\frac{\mu(x1)}{x1}, \frac{\mu(x2)}{x2}, \dots, \frac{\mu(x2)}{x2}\right\}$.

Time Series

Time series is a set of ordered data observations in time. (Hanke & Wichren, 2005). There are four types of data time series, namely horizontal, trend, seasonal and cyclical. The horizontal data is an unexpected and characteristic event random, but the occurrence can affect time series data fluctuation. The trend data is a trend of direction data in the long term, it can increase or decrease. Seasonal data is fluctuations in data that occur periodically. Then the cyclical data is a fluctuation of data more than one year.

Fuzzy Time Series

Fuzzy Time Series is forecasting method that uses fuzzy principles. This method applies fuzy sets to time series analysis. Therefore, fuzzy time series captures past data then uses it to predict future data (Tauryawati & Irawan, 2014). In this method the time series data will be represented into a fuzzy sets. Then the relationship between the data will be defined. Let F(t - 1) = Ai and F(t) = Aj. Relationship between two consecutive observations, F(t) and F(t - 1), referred to as a fuzzy logical relationship (FLR), can be denoted by Ai→ Aj , where Ai is called the left-hand side (LHS) and Aj is the right hand side (RHS) of the FL. After determining the FLR, then the next stage is combine and group the FLR into Fuzzy Logic Relationship Group (FLRG). This FLRG is uses as the basis to determine the forecast value. Then in the finel stage, Defuzzyfication is carried out to obtain the forecast value. There are several ways to determine the forecast value in fuzzy time series, including the model proposed by Chen and Cheng. Calculating the forecast value from defuzzyfication using chen model can be obtained from calculating the mean of the median in each intervals. Let $F(t)$ appropriate with $A1, A2, \dots, An$ and maximum membership value occurs at the interval $u_1, u_2, .., u_n$ where the median is $m_1, m_2, .., m_n$. Then the equation to find the forecast value using Chen model is obtain as:

$$F_t = \frac{m_1 + m_2 + \cdots + m_n}{n}$$

On the other hand, Forecasting using cheng model is slightly different since the grouping of FLR because in the cheng model there is a weighting for each FLR. Then represent these weights into a weighted matrix that has been normalizes $Wn(t)$. Let $L_f = [m_1, m_2, \dots m_k]$ where m_k is the median of each intervals the equation to find the forecast value using cheng model is obtain as

$$F_t = L_f(t-1)W_n(t-1)^T$$

Mean Absolute Percentage Error

Mean Absolute Percentage Error (MAPE) is an error measurement that calculates the percentage deviation between the actual data and forecast value. MAPE calculated by using the average absolute error in each periode divided by the actual observed value in that period.

$$MAPE = \left(\frac{100\%}{n}\right)\frac{\sum_{t=1}^{n}|Xt-F_t|}{Xt}$$

With $X(t)$ is actual data and $F(t)$ is forecast value. The forecasting method has good accuracy if the MAPE value less than 20% (Margi & Pandawa, 2015).

METHODS

Population and Sample

The data used in this study is Indonesia rice productivity from 2001-2021 which wes obtained through Badan Pusat Statistik Indonesia.

Reseach Design

Forecasting method used in this research is fuzzy time series. Two fuzzy time series models will be compared, namely chen and cheng models. Basically the step of fuzzy time series is almost the same. The steps that will be carried out in this study that accordance with forecasting using fuzzy time series method are as follows: step 1 is determine the set of universes. At this stage, looking for the minimum and maximum values of actual data. Then, step 2 is specifies the number and range of classes, This research uses the Sturgess rule to determine the number of intervals divided. Step 3 is determine the fuzzy set against the set universe This stage changes the set of universes that have been divided into a fuzzy sets. Fuzzy sets are formed with the size of the n*n matrix. After that, step 4 is fuzzyfication of historical data. This stage determines the membership value of each fuzzy set from historical data. Step 5 is determine Fuzzy Logical Relationship (FLR). The relationship between two sequentially data, F(t) and F(t-1), becomes F(t-1) → F(t).Next step is determine Fuzzy Logical Relationship Group (FLRG). The value of each relationship obtained will be combined and grouped known as FLRG (Fuzzy Logical Relationship Group). The way of grouping is from the same left side. The final steps of FTS method is defuzzyfication and calculating the forecast value. At this stage, the result of defuzzyfication data will be obtained the forecast value.In this study, two approach models will be used in determining forecast value of the fuzzy time series, that is the model proposed by chen and cheng. The next stage is determined the accuracy of the forecast result using MAPE value. The smaller MAPE value obtained, the more accurate forecast value is. At the end, compare the MAPE value from the forecasting result using chen and cheng models. Then determine which model is more appropriate to forecast Indonesia rice productivity.

RESULT AND DISCUSSION

The actual data used in this research is Indonesia rice productivity in 2001-2021 obtained from BPS Indonesia.

Table 1
Indonesia Rice Productivity

Year	Rice Productivity (ku/ha)	Year	Rice Productivity (ku/ha)
2001	43,88	2012	51,36
2002	44,69	2013	51,52
2003	45,38	2014	51,35
2004	45,36	2015	53,41
2005	45,74	2016	52,36
2006	46,2	2017	51,65
2007	47,05	2018	51,92
2008	48,94	2019	51,14
2009	49,99	2020	51,28
2010	50,15	2021	52,26
2011	49,8		

Data anlyzes

Indonesia rice productivity forecasting will be carried out using the chen and cheng fuzzy time series method as described above. Set of universes, the minimum and maximum values of the actual data to be studied are sought. Set U=[Umin, Umax] where Umin= Xmin-d1 and Umax=Xmax+d2, Xmin is minimum value of the actual data and Xmax is the maximum value of the actal data with d1, d2 is any numbers. Based on the Table 1 is obtained that Xmin=43,88 and Xmax= 53,41. Respectively, let d1=0,88 and d2=1,59 the we get universes U=[43, 55]. Then, Specifies the number and range of classes. This research uses the sturgess rule to determine the number of intervals divided. Partition the univereses into intervals $u_1, u2, \ldots, u_n$ of equal length. Partitioning U into six equal intervals as u_1=[43,45], u_2=[45,47], u_3=[47, 49], u_4=[49,51], u_5=[51,53]. u_6=[53,55]

The next steps is fuzzification. This steps is changes the set of universes that have been divided and are still a set of crips numbers into a fuzzy set based on intervals. Fuzzy sets are formed with the size of the nxn matrix. The value of n is the value obtained from the results of the universe of discourse. we obtained five equal intervals so n=6. Therefore we get 6x6 matrix of fuzzy sets as follows:

Table 2
Matrix of Fuzzy Sets

	A1	A2	A3	A4	A5	A6
A1	1,0	0,5	0,0	0,0	0,0	0,0
A2	0,5	1,0	0,5	0,0	0,0	0,0
A3	0,0	0,5	1,0	0,5	0,0	0,0
A4	0,0	0,0	0,5	1,0	0,5	0,0
A5	0,0	0,0	0,0	0,5	1,0	0,5
A6	0,0	0,0	0,0	0,0	0,5	1,0

We can also represent the fuzzy set above in the following equention:

$$A1 = \frac{1}{U_1} + \frac{0{,}5}{U_2} + \frac{0}{U_3} + \frac{0}{U_4} + \frac{0}{U_5} + \frac{0}{U_6}$$
$$A2 = \frac{0{,}5}{U_1} + \frac{1}{U_2} + \frac{0{,}5}{U_3} + \frac{0}{U_4} + \frac{0}{U_5} + \frac{0}{U_6}$$
$$A3 = \frac{0}{U_1} + \frac{0{,}5}{U_2} + \frac{1}{U_3} + \frac{0{,}5}{U_4} + \frac{0}{U_5} + \frac{0}{U_6}$$
$$A4 = \frac{0}{U_1} + \frac{0}{U_2} + \frac{0{,}5}{U_3} + \frac{1}{U_4} + \frac{0{,}5}{U_5} + \frac{0}{U_6}$$
$$A5 = \frac{0}{U_1} + \frac{0}{U_2} + \frac{0}{U_3} + \frac{0{,}5}{U_4} + \frac{1}{U_5} + \frac{0{,}5}{U_6}$$
$$A6 = \frac{0}{U_1} + \frac{0}{U_2} + \frac{0}{U_3} + \frac{0}{U_4} + \frac{0{,}5}{U_5} + \frac{1}{U_6}$$

After defining the fuzzy sets against the universes, then fuzzyfication the actual data. Determines the membership value of each fuzzy set from actual data. The results of fuzzyfication shown in the following Table 3.

Table 3
Fuzzyfication of Actual Data

Year	Indonesia Rice Productivity (ku/ha)	Fuzzyfication	Year	Indonesia Rice Productivity (ku/ha)	Fuzzyfication
2001	43,88	A1	2012	51,36	A5
2002	44,69	A1	2013	51,52	A5
2003	45,38	A2	2014	51,35	A5
2004	45,36	A2	2015	53,41	A5
2005	45,74	A2	2016	52,36	A6
2006	46,2	A2	2017	51,65	A5
2007	47,05	A3	2018	51,92	A5
2008	48,94	A3	2019	51,14	A5
2009	49,99	A4	2020	51,28	A5
2010	50,15	A4	2021	52,26	A5
2011	49,8	A4			

One of the important parts in the fuzzy time series method is determine the relation between sequential data. The relationship between sequential data that has been converted into fuzzy set is called a Fuzzy Logical Relationship (FLR). Determining the relationship is by rationalize the fuzzy logical relationship, for example, and there are F(i) = Ai dan F(i + 1) = Aj . The relationship between two consecutive observations, F(i) and F(i + 1), becomes F(i) → F(t + 1), is called the fuzzy logic relation, denoted by $Ai \rightarrow Aj$, where Ai is named with LHS (Left Hand Side) or current data and Aj is named with RHS (Right Hand Side) or subsequent data.

Table 4
Fuzzy Logical Relationship (FLR)

Year	Indonesia Rice Productivity (ku/ha)	Fuzzyfication	FLR	
2001	43,88	A1		
2002	44,69	A1	A1→	A1
2003	45,38	A2	A1→	A2

Year	Indonesia Rice Productivity (ku/ha)	Fuzzyfication	FLR	
2004	45,36	A2	A2→	A2
2005	45,74	A2	A2→	A2
2006	46,2	A2	A2→	A2
2007	47,05	A3	A2→	A3
2008	48,94	A3	A3→	A3
2009	49,99	A4	A3→	A4
2010	50,15	A4	A4→	A4
2011	49,8	A4	A4→	A4
2012	51,36	A5	A4→	A5
2013	51,52	A5	A5→	A5
2014	51,35	A5	A5→	A5
2015	53,41	A6	A5→	A6
2016	52,36	A5	A6→	A5
2017	51,65	A5	A5→	A5
2018	51,92	A5	A5→	A5
2019	51,14	A5	A5→	A5
2020	51,28	A5	A5→	A5
2021	52,26	A5	A5→	A5

After getting the FLR, the next step is combine and group FLR. The value of each relationship obtained will be combined or commonly known as FLRG (Fuzzy Logical Relationship Group). The way of grouping is from the same left side.

Table 5
Fuzzy Logical Relationship Group (FLRG)

Group	FLR	FLRG
Group1	A1	A1→A1, A1→A2
Group2	A2	A2→A2, A2→A3
Group3	A3	A3→A3, A3→A4
Group4	A4	A4→A4, A4→A5
Group5	A5	A5→A5, A5→A6
Group6	A6	A6→A5

The next step is defuzzification to get the forecast value. At this stage there is a different ways to obtain the forecast value using chen or cheng model. Therefore we will find the forecast value using Chen model. As described above we need to find the middle value of each interval.

Table 6
Median

		Interval	Median
A1	43	45	44
A2	45	47	46
A3	47	49	48
A4	49	51	50
A5	51	53	52
A6	53	55	54

Forecast value using chen model is obtained by calculate the mean of median based on the FLRG. Then the forecast value is obtained as in Table 7.

Table 7
Forecast Value Using Chen Model

Current State	Next State	Forecast Value
A1	A1, A2	45
A2	A2, A3	47
A3	A3, A4	49
A4	A4, A5	51
A5	A5, A6	53
A6	A5	52

Based on the result in Table 7, we can get forcasting value of Indonesia rice productivity using chen model as shown Table 8.

Table 8
Indonesia Rice Productivity Forecasting Using Chen Model

Year	Indonesia Rice Productivity (ku/ha)	Chen Forecasting value	Year	Indonesia Rice Productivity (ku/ha)	Chen Forecasting value
2001	43,88		2012	51,36	51
2002	44,69	45	2013	51,52	53
2003	45,38	45	2014	51,35	53
2004	45,36	47	2015	53,41	53
2005	45,74	47	2016	52,36	52
2006	46,2	47	2017	51,65	53
2007	47,05	47	2018	51,92	53
2008	48,94	49	2019	51,14	53
2009	49,99	49	2020	51,28	53
2010	50,15	51	2021	52,26	53
2011	49,8	51	2022	-	53

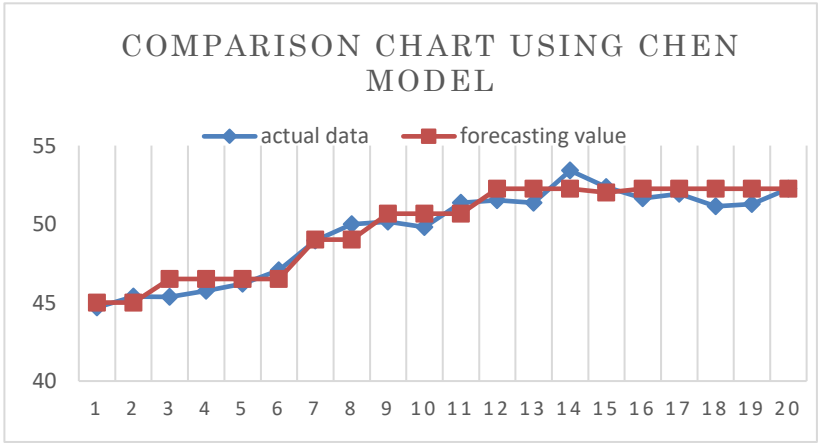

Figure 1
Comparion Chart Between The Actual Data Vs Forecast Value Using Chen Model

The next step is to calculate forecast value using cheng model. To obtain forecasting value of cheng model we need to do the weighting. After the weighting is obtained, we can get the forecast value of Indonesia productivity rice using cheng model shown in Table 9

Table 9
Indonesia Rice Productivity Forecasting Using Cheng Model

Year	Indonesia Rice Productivity (ku/ha)	Cheng Forecasting value	Year	Indonesia Rice Productivity (ku/ha)	Cheng Forecasting value
2001	43,88	45	2012	51,36	50,66667
2002	44,69	45	2013	51,52	52,25
2003	45,38	46,5	2014	51,35	52,25
2004	45,36	46,5	2015	53,41	52,25
2005	45,74	46,5	2016	52,36	52
2006	46,2	46,5	2017	51,65	52,25
2007	47,05	49	2018	51,92	52,25
2008	48,94	49	2019	51,14	52,25
2009	49,99	50,66667	2020	51,28	52,25
2010	50,15	50,66667	2021	52,26	52,25
2011	49,8	45	2022	-	52,25

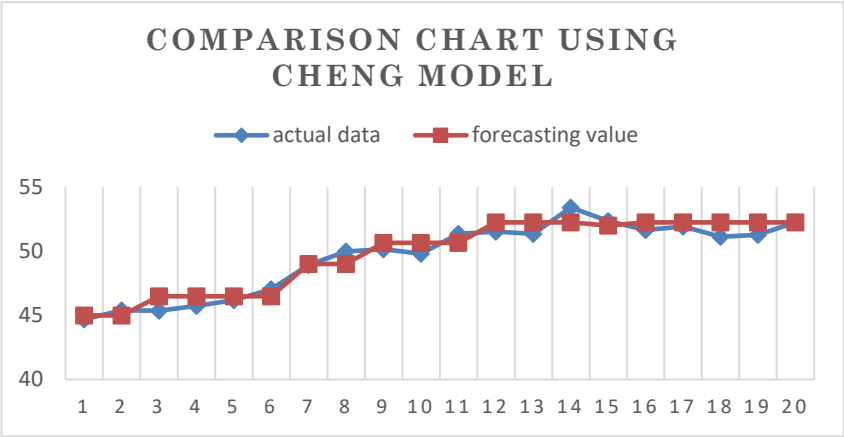

Figure 2
Comparison Chart Between The Actual Data Vs Forecast Value Using Chen Model

The last step in analyzing data is calculate the MAPE value to see the accuracy of forecasting method. A method that get smaller MAPE value means that method is better to forecast Indonesia prodictivity rice. The calculations results based on the forecast value, the MAPE value obtained using chen model is 18% and using cheng model is 12%. It means the MAPE value using cheng model is smaller than chen model.

CONCLUSION AND IMPLICATION

Conclusion

Based on the result of data analysis, it was found that the MAPE value using cheng model is smaller than using chen model. It means that the cheng model is more appropriate used in forecasting data of Indonesia productivity rice. However, the chen and cheng models both give good forecasting result because the MAPE value is less than 20%.the chen and chengs models provides forecasting value that are quite accurate. So

that this method can be used as a reference for government to forecast rice productivity in determining rice import policies

Implication

There are many fuzzy time series models to forecast data, so that for the further research more fuzzy time series models can be compared to give more accurate results in forecasting Indonesia rice productivity.

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