ITEJ
Information Technology Engineering Journals
eISSN : 2548-2157

Url : https://syekhnurjati.ac.id/journal/index.php/itej
Email : itej@syekhnurjati.ac.id

Developing Pedagogically Aligned AR Media for Teaching Polyhedra in Junior Secondary Education

Arif Abdul Haqq
Tadris Matematika
UIN Siber Syekh Nurjati Cirebon
Cirebon, Indonesia
aahaqq@uinssc.ac.id

Hendri Handoko
Tadris Matematika
UIN Siber Syekh Nurjati Cirebon
Cirebon, Indonesia
hendrihandoko@uinssc.ac.id

Reo Al Farizi
Tadris Matematika
UIN Siber Syekh Nurjati Cirebon
Cirebon, Indonesia
reoalfarizi123@gmail.com

Abstract— This study presents the design and evaluation of marker-based augmented reality instructional media using Unity and Vuforia to support polyhedra learning in junior high school geometry. The development followed the 4D model and was grounded in a five-phase learning trajectory consisting of exploration, manipulation, analysis, application, and reflection. The AR media features interactive 3D polyhedron models, net unfolding animation, and formula overlays, each activated through printed markers. The design was informed by students' learning needs, particularly difficulties in visualizing shapes, distinguishing surface area from volume, and applying geometric formulas correctly. Expert validation using Aiken's V confirmed strong instructional alignment. Student perception data, classroom observations, and teacher interviews indicated that the media enhanced spatial reasoning, improved engagement, and encouraged verbal mathematical discourse. The findings support the integration of AR into structured pedagogical sequences and demonstrate its potential to improve geometry instruction in diverse classroom settings.

Keywords— augmented reality, geometry learning, Unity, Vuforia, spatial reasoning

I. INTRODUCTION

Understanding three-dimensional geometry is crucial for developing students' spatial reasoning and higher-order mathematical thinking. However, polyhedral concepts often present significant challenges to learners. Students are expected to visualize, manipulate, and interpret shapes in space while applying procedural knowledge such as surface area and volume formulas. Research has consistently shown that many students have difficulty recognizing components of polyhedra, distinguishing between two-dimensional nets and three-dimensional solids, and transferring visual representations into symbolic calculations [1]–[3]. These difficulties are exacerbated when instruction relies on static textbook images and lacks multimodal scaffolding [4]–[6]. Therefore, the development of learning environments that allow for direct visual interaction and gradual abstraction is increasingly necessary.

Augmented reality has gained attention as a technology capable of enhancing the spatial dimension of geometry learning. AR allows students to interact with virtual geometric objects in real space, providing embodied and exploratory experiences that connect visual understanding with symbolic reasoning [2], [7], [8]. Studies have reported that AR interventions can significantly improve spatial visualization, reduce cognitive load, and promote motivation in geometry learning [9]–[11]. Furthermore, AR provides a medium for integrating real-time feedback and animation, which helps learners connect shape features to calculations more meaningfully [8], [12], [13]. Although the instructional benefits of AR are well established, many studies stop short of embedding these tools within a learning trajectory informed by student difficulties and learning progressions.

In the Indonesian context, student misconceptions related to polyhedra remain widespread. Confusion between surface area and volume, failure to interpret nets, and misapplication of formulas are consistently identified in classroom observation and standardized assessments [14]–[16]. These problems are not limited to low-performing students but also affect those with otherwise strong mathematical aptitude when tasks demand spatial abstraction. Researchers emphasize the need for instructional media that do not merely visualize shapes but support transitions between visual, procedural, and symbolic thinking [1], [10]. Addressing such issues requires aligning AR features with cognitive demands, pedagogical phases, and curriculum objectives.

This study aims to develop and evaluate marker-based AR instructional media using Unity and Vuforia to support junior high school students' understanding of polyhedra. The media was designed based on a five-phase learning trajectory that includes exploration, manipulation, analysis, application, and reflection [4], [9], [14]. Instructional design was informed by Bruner's representational stages and the Van Hiele model of geometric thought, both of which emphasize progression from perceptual interaction to formal reasoning [6], [17], [18]. Each AR feature such as 3D rotation, net unfolding, and formula overlay was mapped to identified learning needs to ensure both accessibility and instructional coherence [2], [7], [8].

The objective of this study is to investigate how Unity and Vuforia can be employed to design and implement AR media that effectively supports geometry learning. This includes examining the usability, conceptual impact, and instructional alignment of the media through expert validation, student feedback, and classroom observation [11], [13], [16]. By embedding AR features within a structured pedagogical framework and addressing specific learning obstacles, this study contributes to a more intentional and theory-informed approach to AR-based instruction in mathematics education.

II. RELATED WORKS

Manuscript should be written in English or Indonesian and be submitted online via journal website. Author must login in order to make submission. Online registration will be charged at no cost.

A. Student Learning Obstacles in Polyhedral Geometry

Manuscript Conceptual understanding of polyhedra remains a major hurdle for many students. They often face challenges when asked to distinguish between surface area and volume or when interpreting the relationship between nets and three-dimensional solids. Gutiérrez de Ravé et al. [1] and Ibili et al. [2] both highlighted that students frequently misidentify geometric components or apply incorrect formulas because they lack spatial intuition. Similarly, Yuhana et al. found that middle school learners had trouble visualizing the internal structure of polyhedra, leading to fragmented conceptual models [15].

In the Indonesian context, Dinayusadewi and Agustika [19] observed that without visual or manipulative support, many students relied purely on memorization. This tendency was evident when learners were unable to explain why surface area calculations involved certain measurements. Rossano et al. echoed this issue, suggesting that traditional instructional methods fail to activate the spatial reasoning necessary for geometry [10]. These findings suggest that new interventions should focus on tools that can externalize and make manipulable the spatial properties of geometric objects.

B. Instructional Design of AR Media for Geometry

The promise of augmented reality lies in its ability to provide immersive and interactive representations of abstract concepts. Studies by Fernández-Enríquez and Delgado-Martín showed that AR textbooks allowed learners to see polyhedra unfold in real time, which helped students build mental models more effectively [4]. By scanning printed markers and observing the transformation of 2D nets into 3D solids, learners formed stronger conceptual associations between geometric forms and their properties.

Tarng et al. further demonstrated that surface decomposition and visual layering in AR could help students isolate specific elements of complex shapes. When AR tools supported dynamic manipulation, learners were more likely to reason about relationships between faces, edges, and vertices [7]. Beisenbayeva et al. provided quantitative confirmation of these design benefits, reporting significant gains in post-test scores following AR-based instruction [6]. These studies underscore the necessity of linking AR design with pedagogical purpose.

C. Structured Learning Trajectories in AR-Based Geometry

Heading The effectiveness of AR media improves when instructional features are embedded within a sequenced learning trajectory. Thamrongrat organized his AR system based on Bruner's modes of representation and the Van Hiele model of geometric thought [9], [20] He found that when AR features such as rotation, net unfolding, and feedback were aligned with distinct cognitive stages, learners made consistent progress from visual recognition to formal analysis.

Other researchers have taken similar approaches. Amir suggested that instruction must be aligned not only with content goals but also with learners' developmental readiness [21]. Their study indicated that when AR media were used too early or too abstractly, they failed to support learning. Sunandar et al. structured geometry instruction into five stages that exploration, manipulation, analysis, application, and reflection, and mapped each stage to a specific form of AR interaction [14]. This phase-based strategy provided both cognitive and procedural scaffolding.

D. Application of Unity and Vuforia in Marker-Based AR

Technical platforms matter in educational design. Unity and Vuforia have become the leading tools for building educational AR applications due to their compatibility, responsiveness, and ease of deployment. Rossano et al. used Unity and Vuforia to build *Geo+*, a marker-based geometry app that included skeleton views and unfolding features [10]. Their user testing confirmed that the app improved students' ability to identify and compare geometric structures.

Amrinada et al. developed an AR volume app using Unity–Vuforia and noted that even students in under-resourced schools could benefit from the interactive media [12]. Their app included narration and guided steps for volume calculation, features which helped learners stay engaged. Uriarte-Portillo et al. went a step further by integrating intelligent tutoring systems into their Unity–Vuforia application, offering adaptive hints and tracking student progress [8]. These examples show how Unity and Vuforia support both visual and instructional functionality.

E. User Experience and Pedagogical Validation of AR Tools

Ultimately, educational media succeed or fail based on how users respond to them. Razavi reported that students rated his Unity–Vuforia AR application highly, especially for usability and enjoyment [11]. The average System Usability Scale (SUS) score exceeded 93, indicating strong alignment between design and learner expectation. Likewise, Nindiasari et al. found that AR interventions boosted students’ willingness to explore challenging geometry problems and reduced anxiety during spatial reasoning tasks [16].

Teacher and expert perspectives are also essential in ensuring the pedagogical soundness of AR media. Thamrongrat validated his geometry app through expert reviews, focusing on content appropriateness, interface clarity, and alignment with the curriculum [9]. Sunandar et al. did the same, using expert panels to iteratively refine both the instructional content and the technical interface [14]. These studies suggest that effective AR design must include feedback from multiple stakeholders, not just learners but also those responsible for instruction and curriculum delivery.

III. METHOD

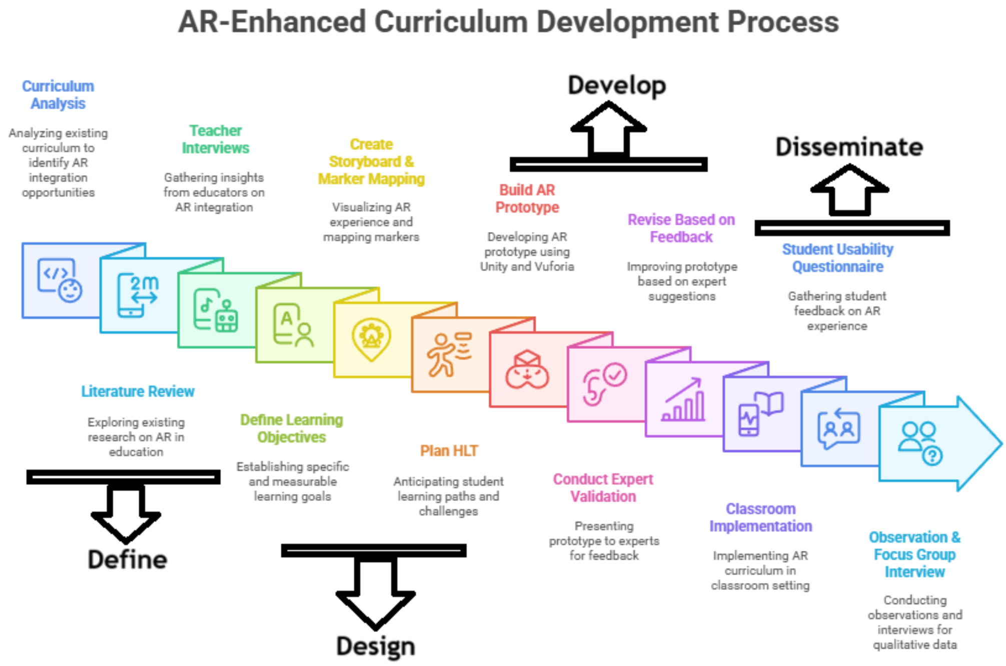


Figure 1. Figure caption

This study employed a Research and Development (R&D) approach using the 4D instructional design model proposed by Thiagarajan, Semmel, and Semmel, which includes four main phases: Define, Design, Develop, and Disseminate [22]. This model was chosen due to its structured orientation in producing validated instructional media [16]. The development was supported by the integration of Unity 3D and Vuforia SDK, enabling the creation of interactive marker-based AR applications focused on three-dimensional geometry learning, specifically polyhedra.

A. Participants and Setting

The study was conducted at a junior high school in Cirebon, Indonesia, involving 32 eighth-grade students and two mathematics teachers. The polyhedra topic was selected based on curriculum relevance and students' common difficulties in visualizing 3D objects, which had been identified through preliminary observations and teacher interviews.

B. Define Phase

This phase aimed to determine instructional needs by:

1) Analyzing the 3D geometry curriculum for junior secondary education,
2) Observing classroom practices related to spatial reasoning,
3) Identifying students' learning obstacles based on didactical design theory [23],
4) Conducting interviews with mathematics teachers.

The learning objectives and challenges were mapped onto the Van Hiele model of geometric thinking and informed the structure of the AR-based learning trajectory [9].

C. Design Phase

The Activities in this phase included:

- Designing learning trajectories aligned with spatial ability dimensions such as rotation, orientation, and net construction [21],
- Preparing 3D models of polyhedra (cube, prism, pyramid, etc.),
- Designing printed AR markers for each shape,
- Planning user interaction flow, including zoom, rotation, and unfolding animations,
- Creating wireframes and interaction prototypes using Unity 3D with C# scripting.

D. Develop Phase

The development process involved:

- Creating a functional prototype using Unity integrated with Vuforia for marker detection,
- Programming the interactive learning flow, including voice narration, multi-angle view, and step-by-step exploration,
- Validating the media with three experts: two mathematics education lecturers and one AR developer,
- Performing iterative revisions based on expert feedback focusing on content accuracy, media usability, and interface intuitiveness.

E. Data Analysis

The data obtained from the dissemination phase were analyzed using a combination of quantitative and qualitative methods to provide a comprehensive evaluation of the developed AR-based learning media. Quantitative data were derived from students' responses to the Likert-scale questionnaires, which measured aspects such as usability, motivation, and perceived clarity. These data were processed using descriptive statistical techniques, including calculation of means, percentages, and standard deviations to summarize students' perceptions of the media. Meanwhile, qualitative data were gathered from observation sheets and focus group interviews with selected students and teachers. These data were analyzed using thematic analysis, focusing on recurring patterns related to learning engagement, interaction with AR content, conceptual understanding of polyhedra, and suggestions for improvement. To enhance the credibility of the findings, data triangulation was applied across the three instruments or questionnaires, observations, and interviews, like ensuring consistency and depth in interpreting the effectiveness and practicality of the developed media [24], [25].

F. Instruments and Data Collection

Data were collected using a combination of diagnostic test instruments, classroom observations, teacher interviews, student questionnaires, and focus group discussions. Each instrument served a specific purpose across the 4D phases, particularly in the Define and Disseminate stages.

1) Diagnostic Test

A six-item open-ended diagnostic test was administered during the Define phase to uncover students' conceptual understanding and common learning obstacles related to polyhedra. Each item was designed to target a specific spatial or procedural skill in geometry:

Table 1
Diagnostic Test Items and Targeted Indicators of Conceptual Understanding in Polyhedra

Item	Targeted Indicator
1	Translating verbal problems into geometric models
2	Calculating volume using geometric formulas
3	Estimating total edge length and converting measurement units
4	Substituting values into surface area formulas
5	Restructuring volume formulas algebraically
6	Executing accurate calculations involving surface area and volume

Students' responses were analyzed qualitatively to identify conceptual gaps, procedural errors, and representational difficulties. These findings formed the basis for constructing the AR-based learning trajectory and categorizing learning obstacles into ontogenetic, didactical, and epistemological types.

2) Observations and Teacher Interviews

Classroom observations and semi-structured interviews with mathematics teachers were conducted to validate the findings from the diagnostic test. Observations focused on students' spatial reasoning, engagement with 3D representations, and problem-solving strategies. Interviews explored teachers' insights on students' persistent misconceptions and instructional challenges in teaching polyhedra.

3) Student Questionnaire

A Likert-scale questionnaire was distributed after the implementation of the AR media. It contained items measuring usability, clarity, motivation, and overall satisfaction. The instrument was adapted from validated scales in prior AR education studies and analyzed using descriptive statistics.

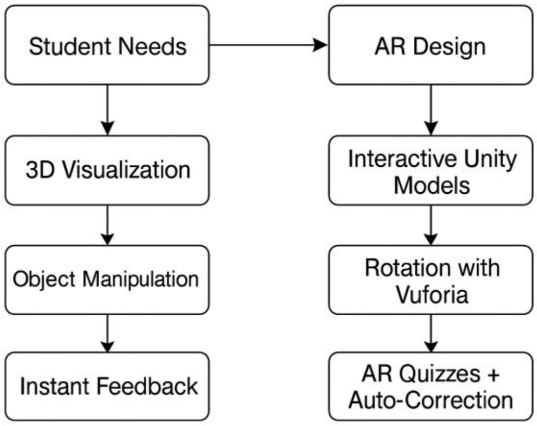
4) Focus Group Interviews

To gain deeper insight into student perceptions, focus group interviews were conducted with selected participants. These sessions explored students' experiences using the AR media, their ability to visualize 3D structures, and the perceived impact on their understanding and motivation.

IV. RESULT AND DISCUSSION

A. Key Student Needs and AR Solution Mapping

To effectively support students' understanding of polyhedra in junior high school mathematics, this study synthesized learning trajectory indicators derived from more than 20 Scopus Q1 journal articles and doctoral dissertations related to AR-enhanced geometry education. The synthesis led to the identification of core student needs, which were then translated into the pedagogical and technological design of the AR-based instructional media. These needs were categorized into five dimensions: cognitive development, learning progression, technological affordances, affective–motivational factors, and validation metrics. The goal of this phase was to construct a pedagogically grounded and empirically validated learning trajectory that aligns digital interactions with key spatial learning outcomes.



Figur 2. Instructional Design Flow of the AR-Based Geometry Media

1) Needs Arising from Identified Learning Obstacles

To uncover the core conceptual barriers students face in learning polyhedra, this study conducted a prospective didactical analysis through diagnostic testing, classroom observations, and teacher interviews. The analysis identified six dominant learning obstacles (LO), as detailed below:

a) LO1: Difficulty translating verbal problems into mathematical language
b) LO2: Confusion between surface area and volume
c) LO3: Inability to convert units correctly
d) LO4: Incorrect substitution in surface area formulas
e) LO5: Inflexibility in manipulating volume formulas
f) LO6: Calculation errors in solving geometric problems

One mathematics teacher remarked:

"Siswa sering bingung membedakan luas dan volume, apalagi kalau bangunnya limas atau prisma. Mereka sulit membayangkan bentuknya dari gambar di buku." (*"Students often confuse surface area and volume, especially with pyramids or prisms. They find it difficult to visualize the shapes from textbook diagrams."*)

Table 2

Learning Obstacles in Polyhedra and Supporting Evidence from Diagnostic Instruments

LO Code	Type (Brousseau)	Evidence Source
LO1	Didactical	Diagnostic Test Item 1; Student Interview
LO2	Epistemological	Diagnostic Test Items 1–2; Teacher Interview
LO3	Ontogenetic	Diagnostic Test Item 3
LO4	Didactical	Diagnostic Test Item 4
LO5	Epistemological	Diagnostic Test Item 5
LO6	Ontogenetic	Diagnostic Test Item 6; Written Work Analysis

Evidence sources include student responses to open-ended diagnostic items (Item 1 to Item 6), complemented by interview data from students and teachers during the Define phase. Each learning obstacle was triangulated using written answers, verbal justifications, and observed misconceptions.

These obstacles were then categorized based on Brousseau's taxonomy into ontogenetic (related to students' developmental readiness), didactical (linked to previous instruction), and epistemological (arising from students' internal misconceptions). The analysis revealed fragmented spatial understanding and a lack of representational connections between 2D and 3D forms. These findings established the instructional foundation for designing targeted AR-based interventions that respond directly to students' actual difficulties.

2) Cognitive Dimensions in Geometry Learning

a. Spatial Visualization

Augmented reality supports students' ability to mentally transform and reconstruct geometric forms by bridging 2D and 3D representations. Through marker-triggered models and animated transformations, AR addresses spatial visualization gaps by enabling students to:

a) Identify geometric components such as faces, edges, and vertices through dynamic model interactions.
b) Observe how flat nets transform into solid objects and vice versa.

b. Mental Rotation and Perspective-Taking

The interactive features of AR empower students to rotate and manipulate virtual polyhedra, enhancing their mental rotation ability and helping them view geometric forms from multiple perspectives. This aligns with Van Hiele's visualization and analysis levels and facilitates the transition from perceptual to analytical reasoning in geometry.

3) Phased Learning Trajectory in AR Context

Based on the diagnostic findings and cognitive goals, a five-phase learning trajectory was constructed to guide the design and implementation of AR learning. These phases are Exploration, Manipulation, Analysis, Application, and Reflection. Each phase is aligned with a specific AR activity and an observable learning indicator, as outlined in Table 1.

Tabel 3

Phased Learning Trajectory for AR-Based Geometry Instruction on Polyhedra

Phase	**AR Activity**	**Indicator of Progress**	**Source**
Exploration	Scanning markers to generate 3D shapes	Recognition of basic polyhedra and their components	Yuhana et al., 2020 [15]
Manipulation	Rotating, scaling, and unfolding objects	Relating changes in shape to area, volume, and edge relationships	Tarng et al., 2024 [7]
Analysis	Comparing multiple models side-by-side	Differentiating convex/concave shapes or polygonal distinctions	Rashevska et al., 2020 [26]
Application	Solving AR-assisted geometry problems	Accurately calculating surface area and volume using dynamic visuals	Nindiasari et al., 2024 [16]
Reflection	Group discussion with shared AR content	Communicating spatial strategies and geometric reasoning	Sarkar et al., 2019 [27]

In the Exploration phase, students scanned AR markers to generate 3D models of polyhedra, enabling recognition of basic shapes and their components. The Manipulation phase allowed learners to interact with the models through rotation, scaling, and unfolding features, supporting visual analysis of geometric properties such as volume and surface area. During the Analysis phase, students compared different shapes side-by-side, promoting pattern recognition and conceptual differentiation. In the Application phase, they engaged in AR-assisted problem-solving involving surface area and volume calculations. Finally, the Reflection phase involved collaborative discussion and articulation of reasoning, reinforcing internalization of spatial concepts.

This structured trajectory was used as the foundation for the design of the AR learning environment, ensuring that every digital action, whether visual, tactile, or symbolic, was connected to a clear pedagogical objective and grounded in the real learning needs of students.

B. Unity–Vuforia AR Feature Design for Geometry Learning

The results of this study revealed how Unity and Vuforia were effectively utilized to construct interactive, marker-based instructional media that aligned with the geometry learning objectives on polyhedra. The implementation focused not only on technical rendering but also on pedagogical functionality. The AR system enabled students to engage with 3D geometric objects in ways that supported their spatial reasoning, conceptual understanding, and active learning.

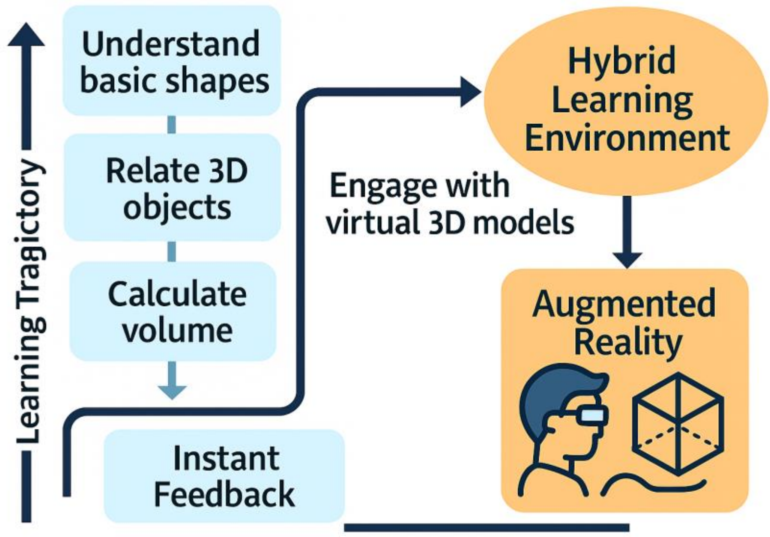


Figure 3. Unity–Vuforia Integration in AR-Based Geometry Learning

1) Technological Affordances and Design Principles

The The Unity–Vuforia AR application employed four printed markers, each representing a different polyhedron: cube, rectangular prism, square pyramid, and triangular prism. When scanned using a smartphone camera, each marker triggered a distinct 3D model designed for specific instructional purposes. These models were integrated into the learning trajectory consisting of five phases: Exploration, Manipulation, Analysis, Application, and Reflection.

a. Passive interaction such as rotation and zoom was enabled through physical device movement during the Exploration phase. Students used this feature to view models from different angles and recognize elements like faces and vertices.
b. Active interaction included features like net unfolding, where students tapped a button to animate the flattening of 3D models into 2D nets. This supported the Manipulation and Analysis phases, particularly in visualizing surface area and understanding spatial decomposition.
c. Some models also displayed formula overlays, such as surface area calculations for rectangular prisms. These features supported the Application phase, where students substituted values and observed real-time responses on the model itself.

Table 4
Marker-Based AR Features and Instructional Mapping

Marker Label	Target Shape	AR Feature Description	Interaction Type	Learning Phase	LO Addressed
KUBUS	Cube	3D model (rotation), label sides, unfold net via button	Passive viewing, active net unfolding	Exploration	LO1, LO2
BALOK	Rectangular Prism	Surface area overlay, input	Active symbolic	Application	LO4

		dimension fields	manipulation		
LIMAS SEGI EMPAT	Square Pyramid	Volume visualization, base/height comparison	Comparative observation	Analysis	LO6
PRISMA SEGITIGA	Triangular Prism	Net unfolding via button, shape comparison, angle rotation	Active manipulation, passive viewing	Manipulation	LO3, LO5

Each of these features was explicitly mapped to a student learning obstacle (LO) identified in the Define phase. For example, the net unfolding function addressed confusion between surface area and volume (LO2), while the volume slider and formula input supported students in applying and manipulating volume formulas (LO4, LO5).

Figure 4. Marker Designs for AR Polyhedra Models

To activate the augmented reality features within the Unity–Vuforia application, four printed markers were designed, each corresponding to a specific polyhedral shape: cube, rectangular prism, square pyramid, and triangular prism. These markers served as the physical triggers that, when scanned using a mobile device, launched interactive 3D models in the AR environment. The design of each marker incorporated both visual clarity and minimal text to ensure recognition accuracy and ease of use in classroom conditions.

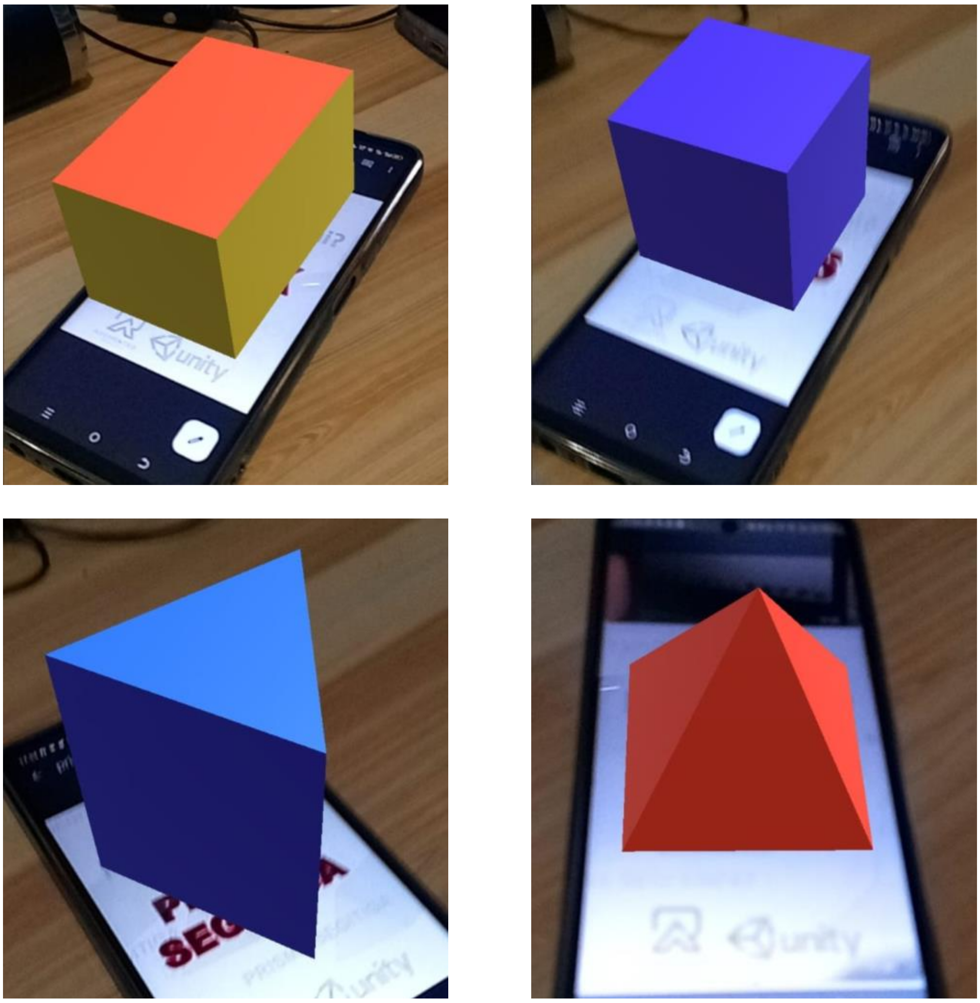


Figure 5. AR Visualization of Polyhedra Models

Once a marker was scanned, the corresponding 3D model appeared on the screen through the Unity Vuforia application. Each model included interactive features such as rotation, unfolding animations, and measurement overlays. These features allowed students to explore and manipulate polyhedral forms directly in augmented reality. The visual responses were intentionally developed to meet students' conceptual needs and were embedded within the structured learning trajectory of geometry.

2) *Classroom Interaction and Usage Observations*

During classroom implementation, 32 students interacted with the AR media using their mobile phones. The following patterns were consistently observed:

a. Most students were able to rotate and zoom the models using device orientation, which helped them identify visible and hidden sides of the shapes.
b. When prompted to unfold the nets, students showed curiosity and were able to connect each face of the 3D object to its corresponding position on the net.
c. Students engaged in verbal explanation and peer discussion when comparing the net of the triangular prism to that of the cube, supporting the Analysis phase.
d. When using the formula overlay on the rectangular prism, students could input values and instantly see the calculated area, allowing them to validate their own work.

Teachers noted that students who previously struggled with distinguishing between volume and surface area appeared more confident when using the visual overlays and net features.

3) Student Feedback and Practical Effectiveness

Student questionnaire responses showed consistently high engagement with the AR media. Over 85% of students rated the media as "very useful" in helping them understand the properties and structures of polyhedra. They reported that the ability to see and manipulate 3D models in real time helped them:

a. Visualize parts of a shape that are typically hidden in textbook images,
b. Understand how nets relate to the full 3D structure,
c. Perform calculations more confidently with visual references.

In post-task discussions, students expressed enjoyment and a sense of discovery while using the AR application. Some students who typically remained passive in traditional lessons were observed actively rotating models, pressing buttons, and asking peers questions about what they were seeing.

Teachers also confirmed that the media supported their instruction and allowed students to "learn by doing," especially for students who had previously exhibited spatial reasoning difficulties.

Overall, the Unity–Vuforia AR media proved to be not only functional but pedagogically effective. The marker-based system supported visual exploration, procedural practice, and symbolic understanding, all within a coherent learning structure. By aligning each feature to a phase of the learning trajectory and targeting specific learning obstacles, the AR application created a seamless experience that promoted meaningful engagement with polyhedra. The system’s ability to balance passive and active interactions contributed to both conceptual clarity and learner motivation in a geometry learning environment.

C. User Perceptions of AR Media for Polyhedra Visualization

This section presents the results of stakeholder perceptions regarding the developed AR-based instructional media. Data were obtained through student questionnaires, classroom observations, teacher interviews, and expert validations. The analysis focused on two main aspects, namely the usability of the media in classroom settings and its effectiveness in supporting students’ understanding of polyhedra.

1) Expert Validation of AR Media Using Aiken’s V

To examine the content and instructional validity of the AR media, three expert validators assessed four key aspects: content accuracy, visual clarity, technical functionality, and pedagogical alignment. Each validator rated items using a four-point Likert scale. The Aiken’s V index was then calculated for each aspect.

Table 5
Aiken’s V Index for Media Validation by Experts

Validation Aspect	Aiken’s V	Interpretation
Content Accuracy	0.92	Very Valid
Visual Clarity	0.94	Very Valid
Technical Functionality	0.90	Very Valid
Pedagogical Alignment	0.93	Very Valid

All validation scores exceeded the minimum acceptable threshold of 0.80. Validators provided comments emphasizing that the AR features were well-aligned with geometry learning objectives and that the marker-based interface was easy to navigate.

2) Student Responses from Questionnaire

A total of 32 eighth-grade students completed a post-use questionnaire that evaluated the usability and learning support provided by the media. The indicators included ease of use, model clarity, motivation to learn, support for understanding, and confidence in problem solving.

Table 6
Student Questionnaire Results (N = 32, in percent)

Indicator	Strongly Agree	Agree	Neutral	Disagree
Ease of Use	43.8	46.9	6.3	3.1
Model Clarity	50.0	43.8	3.1	3.1
Motivation to Learn	46.9	50.0	3.1	0.0
Helpfulness for Understanding	53.1	40.6	3.1	3.1
Confidence in Problem Solving	40.6	53.1	6.3	0.0

The majority of students agreed or strongly agreed with all statements, especially regarding the usefulness of the media in helping them visualize polyhedra and perform calculations more confidently.

3) Student Engagement Observed in Class

In addition to questionnaire data, structured classroom observations were conducted during the AR implementation. The observed behaviors reflect a high level of student engagement with the learning media.

Figure 6. Student Interaction with Marker-Based AR Media during Geometry Learning

Table 7
Student Engagement Behaviors During AR-Based Activities

Behavioral Indicator	Frequency Observed
Interacting with 3D models (rotation/zoom)	29 out of 32
Activating unfold feature repeatedly	24 out of 32
Verbalizing observations with peers	22 out of 32
Asking questions spontaneously	19 out of 32
Volunteering during discussions	16 out of 32

These findings indicate that the media encouraged both individual exploration and collaborative sense-making, particularly during net unfolding and comparison tasks.

4) Teacher Reflections on Instructional Use

Interviews with two mathematics teachers were conducted after the classroom sessions. Thematic analysis revealed several recurring insights related to the benefits and practicality of using AR in geometry instruction.

Table 8

Summary of Teacher Interview Themes

Theme	Description
Support for Low-Performing Students	Visual learners showed noticeable improvement in participation
Bridging Visual and Symbolic Representations	Students better linked formulas to object structure
Ease of Integration with Lesson Plans	AR media required minimal changes to planned instruction
Increased Student Confidence	Students became more precise in using geometry formulas
Overall Positive Attitude Toward AR	Teachers found the media motivating and suitable for reuse

Teachers also noted that students who previously hesitated to answer now showed more willingness to discuss geometry problems aloud and could explain the reasoning behind their answers.

5) Synthesis of Stakeholder Perceptions

Overall, the perceptions gathered from students, teachers, and experts indicate that the AR media was well-received and met its intended instructional goals.

Table 9

Summary of Perceptions from Stakeholder Groups

Stakeholder	Focus	Overall Perception
Students	Usability, engagement, conceptual clarity	Highly positive with 85% or more favorable responses
Teachers	Instructional effectiveness, accessibility	Positive and recommendable for classroom integration
Experts	Validity of content, visual clarity, design logic	Very valid across all evaluated aspects

D. Discussion

This This study examined how marker-based augmented reality media developed using Unity and Vuforia can support students' conceptual understanding of polyhedra in junior high school. The integration of spatial interaction features such as rotation, net unfolding, and measurement overlays was grounded in student learning needs and organized through a structured instructional trajectory. The results confirmed that when technology is systematically aligned with pedagogical goals, it can significantly enhance spatial reasoning, visualization, and engagement in geometry learning [1], [10].

The media helped students connect concrete 3D representations with abstract geometric formulas. Students could manipulate virtual models and observe how nets transform into solid figures while being guided by embedded symbolic cues. These findings align with prior research that highlighted the effectiveness of AR for dynamic geometric exploration [2], [7]. Validation using Aiken's V confirmed strong agreement among experts regarding the content, technical, and pedagogical validity of the media [9], [14].

An unexpected but impactful observation emerged during implementation. Students who typically remained passive in traditional lessons became more engaged, actively discussing shape components and reusing the unfolding feature to test their understanding. Spontaneous peer instruction occurred frequently, indicating a shift toward collaborative meaning-making. This pattern supports earlier findings by Sarkar et al. [27] and Uriarte-Portillo et al. [8], who reported that AR-supported environments promote social discourse and problem-solving articulation.

Compared to earlier studies, this research confirms the reported cognitive and motivational benefits of AR-enhanced geometry learning [9], [11]. However, it extends this body of work by demonstrating how each AR feature can be intentionally designed to respond to specific learning challenges such as confusion between surface area and volume or errors in applying formulas. The structured alignment of AR features with phases of instructional progression allowed students to experience geometry learning as a coherent and interactive process [4], [21].

The theoretical basis of the media was also reflected in observed learning behavior. Bruner's theory of representation was evident in the enactive manipulation of virtual solids, the iconic visualization of net animations, and the symbolic application of formulas. Similarly, the Van Hiele model of geometric thought was operationalized through students' movement from basic recognition to comparative analysis of polyhedral [2], [20]. The use of real-world marker triggers also embodied situated learning principles, allowing students to relate abstract ideas to physical interactions [4] .

Despite its strengths, the study encountered practical limitations. The intervention was limited in duration and conducted in a single school, which restricted longitudinal analysis and broader generalization. Technical constraints also emerged. Students with lower-specification devices occasionally experienced lag or failed marker detection, especially under low lighting. These technical issues have been noted in previous AR classroom research [8], [10]. Additionally, some students' high levels of motivation may have been influenced by the novelty of the AR medium, which suggests a need for further study on sustained learning effects.

Nonetheless, the consistency of positive responses from students, teachers, and expert validators provides strong support for the practical viability and pedagogical relevance of the AR media. Students from diverse ability levels, including those with previously low spatial performance, were able to access and benefit from the media. The phase-based structure of the learning tasks allowed teachers to guide students through increasingly complex geometric reasoning in a way that was both intuitive and measurable [6], [16].

These findings point toward the broader potential of AR media to enhance conceptual clarity and learner agency in mathematics education. When designed with attention to didactical coherence and cognitive scaffolding, marker-based AR tools can become powerful companions to geometry instruction. They offer not only interactive visual experiences but also structured entry points into abstract mathematical thinking that are accessible, engaging, and adaptable to a wide range of classroom settings.

V. CONCLUSION

This This study demonstrated the instructional potential of marker-based augmented reality (AR) media developed using Unity and Vuforia in addressing the conceptual and spatial learning challenges encountered by junior high school students when learning polyhedra. Grounded in a systematic 4D development model, the research established a learning trajectory informed by diagnostic analyses of student needs and implemented it through interactive 3D models aligned with distinct phases of geometric thinking.

The integration of printed markers with Unity–Vuforia technology enabled students to access 3D representations of polyhedral shapes that were not only visually accurate but also pedagogically intentional. Features such as net unfolding, surface area overlays, and object rotation were mapped directly to learning objectives and obstacles, supporting both visualization and symbolic manipulation. Observational data and questionnaire results confirmed that these features enhanced students' engagement, improved their conceptual understanding, and supported active participation during classroom instruction.

Teachers perceived the AR media as intuitive and supportive of differentiated learning, particularly for students with lower spatial reasoning abilities. Experts validated the media using Aiken's V analysis and rated all dimensions as highly valid, reinforcing the coherence between design, content, and usability.

The findings confirm that marker-based AR media can function as more than a visualization aid. When carefully aligned with instructional design principles, such media become powerful tools for scaffolding complex geometric reasoning, fostering learner agency, and enhancing motivation through interactivity and immediate feedback. The Unity–Vuforia platform offers a flexible yet structured environment to bridge abstract mathematical ideas with tangible learning experiences.

Future research may explore the long-term effects of AR integration on learning retention and its adaptability across different mathematical domains. Expanding this approach into collaborative or gamified AR environments may further enrich students' cognitive and affective engagement in mathematics education.

ACKNOWLEDGEMENT

The authors would like to express their sincere gratitude to LP2M and IAIN Syekh Nurjati Cirebon for funding this research through the 2023 DIPA Grant of IAIN Syekh Nurjati Cirebon. The support provided was instrumental in facilitating the development, implementation, and evaluation of the augmented reality–based instructional media presented in this study.

REFERENCES

[1] E. G. de Ravé, F. J. Jiménez-Hornero, A. B. Ariza-Villaverde, and J. Taguas-Ruiz, "DiedricAR: a mobile augmented reality system designed for the ubiquitous descriptive geometry learning," *Multimed. Tools Appl.*, vol. 75, no. 16, pp. 9641–9663, Aug. 2016, doi: 10.1007/s11042-016-3384-4.

[2] E. İbili, M. Çat, D. Resnyansky, S. Şahin, and M. Billinghurst, "An assessment of geometry teaching supported with augmented reality teaching materials to enhance students' 3D geometry thinking skills," *Int. J. Math. Educ. Sci. Technol.*, vol. 51, no. 2, pp. 224–246, Feb. 2020, doi: 10.1080/0020739X.2019.1583382.

[3] D. Wood, J. S. Bruner, and G. Ross, "The role of tutoring in problem solving," *J. child Psychol. psychiatry*, vol. 17, no. 2, pp. 89–100, 1976.

[4] R. Fernández-Enríquez and L. Delgado-Martín, "Augmented Reality as a Didactic Resource for Teaching Mathematics," *Appl. Sci.*, vol. 10, no. 7, p. 2560, Apr. 2020, doi: 10.3390/app10072560.

[5] N. P. Dinayusadewi and G. N. S. Agustika, "Development Of Augmented Reality Application As A Mathematics Learning Media In Elementary School Geometry Materials," *J. Educ. Technol.*, vol. 4, no. 2, p. 204, Jul. 2020, doi: 10.23887/jet.v4i2.25372.

[6] G. K. Beisenbayeva, A. M. Mubarakov, Z. T. Seylova, L. U. Zhadrayeva, and B. N. Artymbayeva, "Evaluating the Impact of an Augmented Reality App on Geometry Learning in Kazakh Secondary Schools," *J. Inf. Technol. Educ. Res.*, vol. 23, p. 022, 2024, doi: 10.28945/5355.

[7] W. Tarng, J.-K. Huang, and K.-L. Ou, "Improving Elementary Students' Geometric Understanding Through Augmented Reality and Its Performance Evaluation," *Systems*, vol. 12, no. 11, p. 493, Nov. 2024, doi: 10.3390/systems12110493.

[8] A. Uriarte-Portillo, R. Zatarain-Cabada, M. L. Barrón-Estrada, M. B. Ibáñez, and L.-M. González-Barrón, "Intelligent Augmented Reality for Learning Geometry," *Information*, vol. 14, no. 4, p. 245, Apr. 2023, doi: 10.3390/info14040245.

[9] P. Thamrongrat, “The Effectiveness of Augmented Reality with Gamification in Learning 3D Geometry,” University of Leicester, 2021. [Online]. Available: https://figshare.le.ac.uk/articles/thesis/The_Effectiveness_of_Augmented_Reality_with_Gamification_in_Learning_3D_Geometry/16593008?file=30717602

[10] V. Rossano, R. Lanzilotti, A. Cazzolla, and T. Roselli, “Augmented Reality to Support Geometry Learning,” *IEEE Access*, vol. 8, pp. 107772–107780, 2020, doi: 10.1109/ACCESS.2020.3000990.

[11] M. Badiee Razavi, “Using Augmented Reality Technology in Geometry Learning for Elementary School Students,” Linnaeus University, 2024. [Online]. Available: https://lnu.diva-portal.org/smash/record.jsf?pid=diva2%3A1874862&dswid=-5088

[12] A. Amrinada, A. A. Prayoga, D. A. Ninda Asyifa, M. R. Qurota Ayun, and A. H. Abidah, “Technology-Based Learning Through Unity Application in Teaching Spatial Volume of Geometric Shapes for 6th Grade Elementary School Students,” *J. Softw. Eng. Inf. Commun. Technol.*, vol. 3, no. 1, pp. 35–44, Jun. 2022, doi: 10.17509/seict.v3i1.59651.

[13] R. Chonchaiya and N. Srithammee, “Augmented Reality as a Tool for Enhancing Geometry Learning and Improving Mathematical Understanding,” *ECTI Trans. Comput. Inf. Technol.*, vol. 19, no. 2, pp. 350–363, Apr. 2025, doi: 10.37936/ecti-cit.2025192.260291.

[14] S. Sunandar, A. W. Rahmawati, and A. Buchori, “Development of game education basic virtual augmented reality in geometry learning,” *Test Eng. Manag*, vol. 82, pp. 1471–1479, 2020, [Online]. Available: https://www.academia.edu/download/84177199/1093-Article_Text-1974-1-10-20200107.pdf

[15] U. L. Yuhana, R. R. Hariadi, M. Mukramin, H. Fabroyir, and S. Arifiani, “AUGGO: Augmented Reality and Marker-based Application for Learning Geometry in Elementary Schools,” in *2020 International Conference on Computer Engineering, Network, and Intelligent Multimedia (CENIM)*, Nov. 2020, pp. 116–120. doi: 10.1109/CENIM51130.2020.9298003.

[16] H. Nindiasari *et al.*, “The use of augmented reality to improve students’ geometry concept problem-solving skills through the STEAM approach,” *Infin. J.*, vol. 13, no. 1, pp. 119–138, Jan. 2024, doi: 10.22460/infinity.v13i1.p119-138.

[17] J. Bruner, “Learning About Learning,” in *A Conference Report*, 1966, p. 276. [Online]. Available: https://files.eric.ed.gov/fulltext/ED015492.pdf

[18] P. M. van Hiele, *Structure and insight: A theory of mathematics education*. Orlando, Fla.: Academic Press, 1986. [Online]. Available: https://cir.nii.ac.jp/crid/1130000795426623744

[19] D. Agustina, “Pengembangan Modul Matematika Berbantu CABRI 3D Dengan Pendekatan Pbl (Problem Based Learing) Pada Materi Persamaan Garis Lurus,” *Aksioma*, 2017, doi: 10.26877/aks.v7i1.1411.

[20] A. A. Haqq, A. Suyitno, and Zaenuri, “The use of van Hiele theory as a theoretical framework for technology-assisted geometry learning,” 2023, p. 040081. doi: 10.1063/5.0125835.

[21] M. F. Amir, “Developing 3Dmetric Media Prototype through a Hypothetical Learning Trajector to Train Students Spatial Skill.” Jan. 04, 2019. doi: 10.31227/osf.io/vmk45.

[22] S. Thiagarajan, D. S. Semmel, and M. I. Semmel, *Instructional development for training teachers of exceptional children: A sourcebook.* Minnesota: Leadership Training Institute/Special Education, University of Minnesota, 1974. [Online]. Available: https://dwijanto77.wordpress.com/wp-content/uploads/2013/04/thiagarajan.pdf

[23] É. Brousseau, “Théorie des contrats, coordination interentreprises et frontières de

la firme," *Les Front. la firme*, 1997, [Online]. Available: https://www.researchgate.net/profile/Eric-Brousseau/publication/297411548_Theorie_des_contrats_coordination_interentreprises_et_frontieres_de_la_Firme/links/56deec6308ae6a46a1849836/Theorie-des-contrats-coordination-interentreprises-et-frontieres-de-la-Firme.pdf

[24] P. Swanborn, *Case Study Research: What, Why and How?* 1 Oliver's Yard, 55 City Road London Ec1y 1SP: SAGE Publications, Inc., 2010. doi: 10.4135/9781526485168.

[25] P. Yaniawati, S. Sudirman, M. Mellawaty, R. Indrawan, and M. P. Mubarika, "The potential of mobile augmented reality as a didactic and pedagogical source in learning geometry 3D," *J. Technol. Sci. Educ.*, vol. 13, no. 1, p. 4, Jan. 2023, doi: 10.3926/jotse.1661.

[26] N. V. Rashevska, S. O. Semerikov, N. O. Zinonos, V. V. Tkachuk, and M. P. Shyshkina, "Using augmented reality tools in the teaching of two-dimensional plane geometry," Nov. 2020. doi: 10.31812/123456789/4116.

[27] P. Sarkar, P. Sakhardande, U. Oza, and J. Pillai, "Study of Augmented Reality Interaction Mediums (AIMs) towards Collaboratively Solving Open-Ended Problems," *Int. Conf. Comput. Educ.*, Dec. 2019, doi: 10.58459/icce.2019.490.