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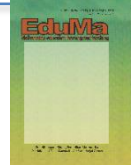
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## Using Geogebra and Manipulative's on Conics as Learning Tool for Student

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### abstract

USING GEOGEBRA AND MANIPULATIVE'S ON CONICS AS LEARNING TOOL FOR STUDENT. Students are required to complete tasks related to eccentricity in the conic. If the eccentricity is changed the value is obtained through different forms of conic. If the eccentricity is equal to zero, then in order to form a special ellipse conic that is a circle. To assist students in learning it required the learning media such as manipulatives and Geogebra software. We find that students generally accomplish tasks very well.

### Keywords:

Analytic geometry, manipulatives, Geogebra, circle, conic.



Open Access

## INTRODUCTION

The use of educational technology can help students in learning. Research by using WebQuest shows that the students generally found the correct problem, and they came up with the correct algebraic solutions and the students drew correct graphs of conic sections (Kurtuluş & Ada, 2012). The use of Geogebra in learning, the visuality increases the students' attention towards the math lessons, which predominantly consist of an abstract concept (Reis & Ozdemir, 2010). Research shows that the learning of circle by the traditional method, without using manipulatives or geometry software, significantly influence student's negative attitude and high student anxiety level (Budiman, 2014). The statistically significant and high overall effect of dynamic geometry software-based instruction on students' mathematical test scores demonstrated that Dynamic Geometry Software intervention should be an effective approach to transforming traditional teaching practices (Chan & Leung, 2014).

The use of manipulatives will assist students in constructing an understanding of mathematical concepts. The research report shows an increase in the mathematical understanding of students who received analytic geometry lectures on conic concepts by using manipulatives higher than mathematical comprehension of students whose lectures were without manipulatives (Sudihartinih & Purniati, 2017). Research studies prove the significance of math manipulatives in assuring a higher and more appreciated form of learning through hands-on education (Gardner, 2016). The use of manipulatives in learning is done by researchers ((Jasmine et al., 2014; Langbort, 2014). The use of

manipulatives has many benefits: manipulatives being assisted in concrete visualization, it is provided hands-on learning, followed by building better understanding, help to engage students, enjoyment and provide motivation, can help children to grasp concepts or reinforce them, can be used to introducing concepts, which are mathematical, provided for opportunities for collaborative learning to occur (Marshall & Swan, 2008). The use of manipulatives can foster positive attitudes toward learning, abstract concepts are present in concrete form, the relationship of abstract concepts and the nature can be understood, so it can be used as research object (Suherman & Dkk., 2003).

Manipulatives as learning aids can serve as real models of concepts so that students can learn them with their five senses, and play a role in decreasing degrees of mathematical abilities (Danoebroto, 2012). He also simply gives an example before learning about the dimension  $n$ , students understand the basic idea through the object model of dimension 2 or dimension 3 so that the process of reconstruction of knowledge through observation, experiment, or other experience.

In this article will be presented student assignments about eccentricity and forms of conic completed using manipulatives and Geogebra

For citation sources, use the following writing format: (Durmuş, 2011) or (Graycar, 1997), or (Kumar Singh, 2006), or (Mhakure & Mokoena, 2011).

## METHODS

The experience of conic learning for students is important because it becomes their provision to teach it in school. The

method of this research is to use the method carried out by Panorkou & Pratt (Panorkou & Pratt, 2016) entitled “Using Google Sketch Up to Develop Students’ Experiences of Dimension in Geometry”, and Kurtulus and Ada (Kurtuluş & Ada, 2012) about “WebQuest on conic sections as a learning tool for prospective teachers”. The participants of this study were second-semester students who studied analytical geometry in the mathematics education department at one of the universities in Indonesia. The number of participants was 32 people (10 men and 22 women). In previous years conic learning did not use Geogebra and manipulative learning. So that this paper will be explained “Using Geogebra and Manipulative’s on Conics as Learning Tool for Student”. At the beginning of learning, students understand the conic preliminary concepts as follows.

Conic is a locus of the points so that the ratio of its distance from a fixed point to its distance from a fixed line is constant. The fixed point is called the focus. The fixed-line is called the directrix. While the constant ratio is called eccentricity and is denoted by  $e$ . If  $e=1$  then the conic is a parabola. If  $e<1$  then the conic is an ellipse. If  $e>1$  then the conic is a hyperbola. A circle is a special ellipse with  $e=0$ .

## RESULT AND DISCUSSION

The following are the steps in conic learning using Geogebra and manipulatives.

### *Task*

Students are asked to analyze eccentricity on conic by varying their values. They are first-degree students of analytic geometry at one of the universities in Indonesia.

Next, the student analyzes the relation between the conic and eccentricity by manipulatives and Geogebra.

*Process.* The task will consist of two parts:

In part I students are asked to analyze the conic in the manipulatives. Perform a demonstration of conic. Look at the change of focus and directive on the manipulatives of each image.

In part II students were asked to analyze the conic in Geogebra. Input eccentricity values on Geogebra. Look at the change of foci and directrix on Geogebra of each figure. Then save the figure of each eccentricity value.

### *Resources*

The first source of learning is manipulatives designed by researchers. Manipulatives made with a size that is not too large, so easy to carry. As for the making step, first, create an ellipse pattern using Geogebra software, then print on color spotlight paper and paste on both plywood boards using the double tip. Second, Put the rafters around the plywood. Third, plug the spikes according to the available patterns and attach the woolen threads to the spikes at the foci. Fourth, attach the plastic ruler to the ID card strap, then plug it into the edge of one of the boards. Fifth, give color to taste on triplex using paint and attach hinge, latch, and doorknob on manipulatives.

The second learning resource of Geogebra software created by Hohenwarter can be freely accessible from [www.Geogebra.org/](http://www.Geogebra.org/). The basic idea of the software Geogebra is to join algebra, geometry, and calculus, which other packages treat separately, into a single easy-to-use package for learning and teaching mathematics from elementary through university level, it is

available free of charge on the Internet, has been translated to 36 languages (Hohenwarter, M. Hohenwarter, Kreis, & Lavicza, 2008).

*Evaluation.* This task will be evaluated using the rubric.

*Conclusion.* Using manipulatives and Geogebra, students solved problems with conic.

Following is the assessment of the students' task. The analysis of eccentricity, foci, and directrixs relationships was done using manipulatives and Geogebra. Here are the tasks assigned to students.

*Part I:* The task of the student identifies the form of conic with the eccentricity, locus of foci and directrixs.

The manipulatives used can be seen in the following 1, 2, dan 3 figures. Students are asked to show for  $e > 1$ . The way of demonstration as follows (Sudihartinih and Purniati, 2016).

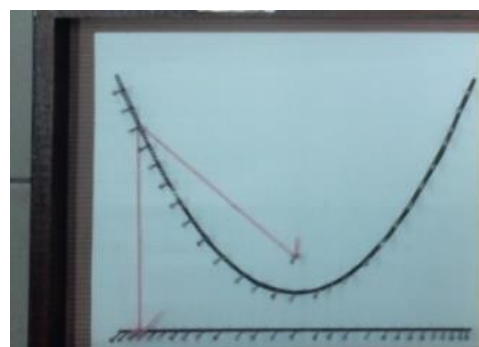


**Figure 1** Hyperbola Manipulatives

Wrap one end of the yarn on the foci. Then wrap it around the spikes on the hyperbola. Then wrap perpendicularly to the spikes on the directrixs. Repeat the same way but through another nail on the hyperbola and directrixs. Calculate the ratio between the distance from the spikes on the hyperbola to the foci and the distance from the spikes on the hyperbola

to the directrixs. This demonstration shows that the ratio of its distance from a fixed point to its distance from a fixed line is constant that is more than one.

Furthermore, students are asked to show for  $e = 1$ . The way of demonstration as follows (Sudihartinih and Purniati, 2016).



**Figure 2** Hyperbolic manipulatives

Wrap one end of the yarn on the focus. Then wrap it back on the spikes on the parabola. Then wrap it back perpendicular to the spikes on the directrix. Repeat the same way but through another nail on the parabola and directrix. This demonstration shows that the ratio of its distance from a fixed point to its distance from a fixed line is constant that equals one.

Students are asked to show for  $e < 1$ . The demonstration is shown in Figure 3.



**Figure 3** Circle Manipulatives Based on Definition of conic

Figure 3 is a manipulative containing, first ellipse A with equations  $\frac{x^2}{100} + \frac{y^2}{75} = 1$ . Secondly, the ellipse B with the equation  $\frac{x^2}{47} + \frac{y^2}{44} = 1$ . Third, the ellipse C with the equation  $\frac{x^2}{49} + \frac{y^2}{37} = 1$ . Fourth, an ellipse D which is a special ellipse in the form of a circle with equations  $\frac{x^2}{25} + \frac{y^2}{25} = 1$ .

Students then use manipulatives to experiment. As for how to use manipulatives are:

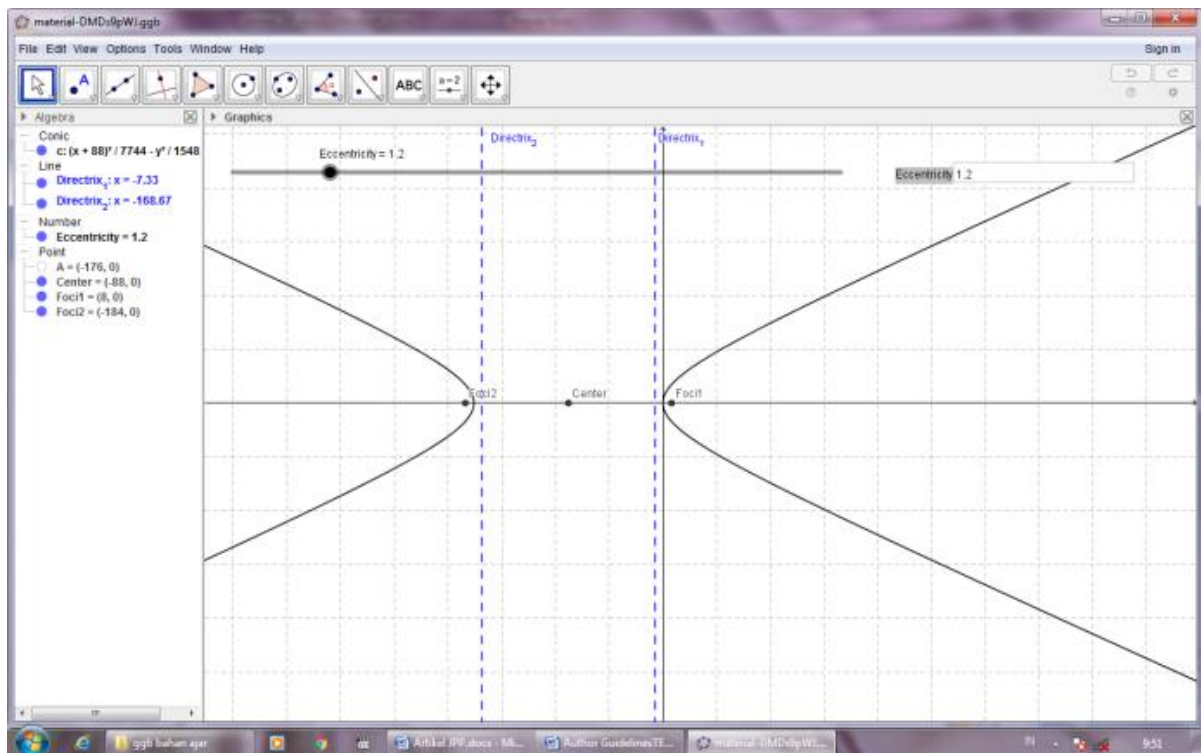
1. Open the manipulatives as shown in figure 9.
2. See the ellipse A, connect the available yarn in the focus to the point on the ellipse, then project the point on the ellipse against the directrix, with the same thread.

3. Compare the distance from the foci to the point on the ellipse, and from the point on the ellipse to the point on the directrix. So we get the value of eccentricity for ellipse A.

4. Perform step at number a and number b for ellipse B and ellipse C.

*Part II:* Students' assignments sketch the Geogebra.

Furthermore, students learn to use Geogebra. Using Geogebra students can be assigned to create the following sketch. For task 1, in Geogebra select  $e = 1.2$ . What images are obtained? Where are the focus and the directrix? The completion of this task is in Figure 4.



**Figure 4** Task 1

For task 2, in Geogebra select  $e = 0$ . What images are obtained? Where are the focus

and the directrix? The completion of this task is in Figure 5.

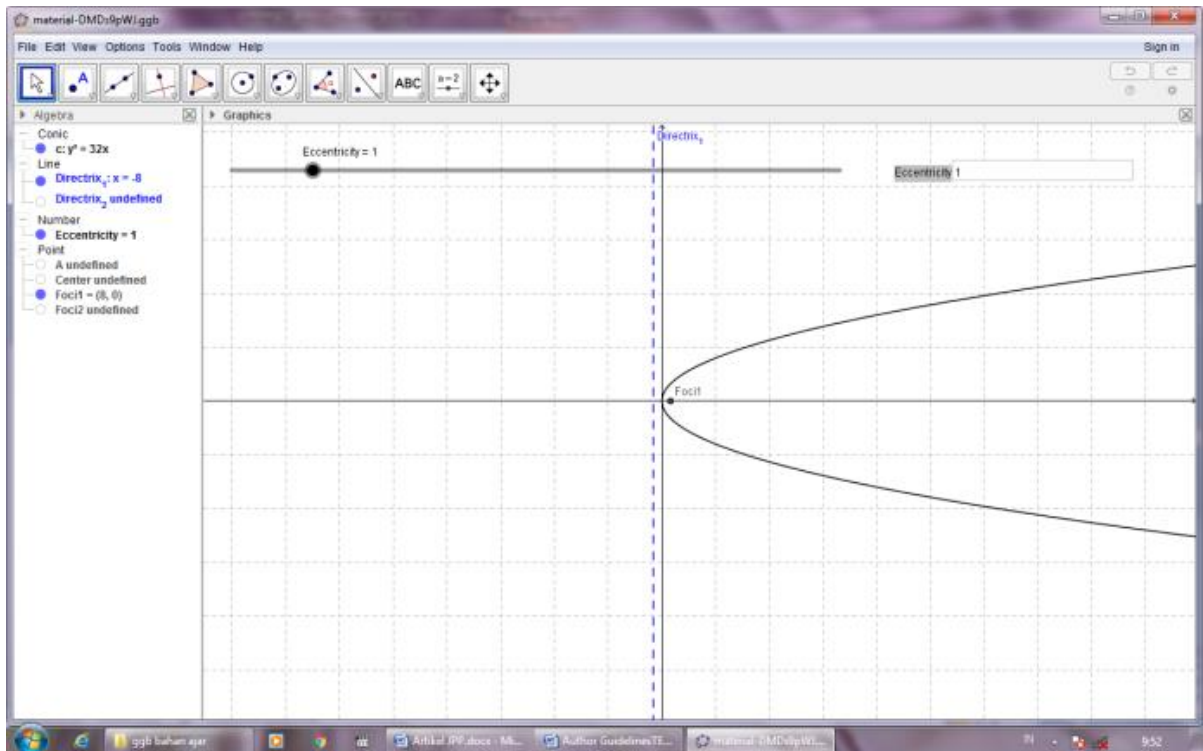


Figure 5 Task 2

For task 3, in Geogebra select  $e = 0.9$ . What images are obtained? Where are the focus and the directrix? The completion of this task is in Figure 6.

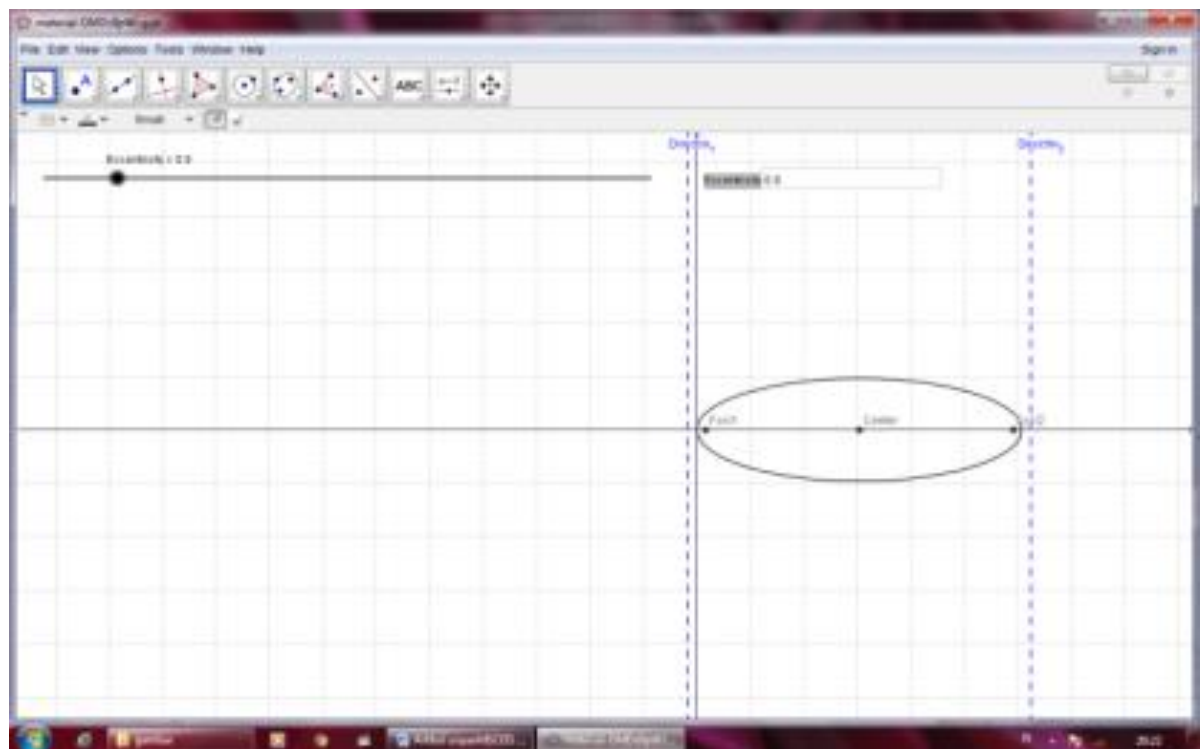


Figure 6 Task 3

On task 4 students sketched the Geogebra with  $e = 0.7$ . What images are obtained? Where are the boards and directrix? The completion of this task is in Figure 7.

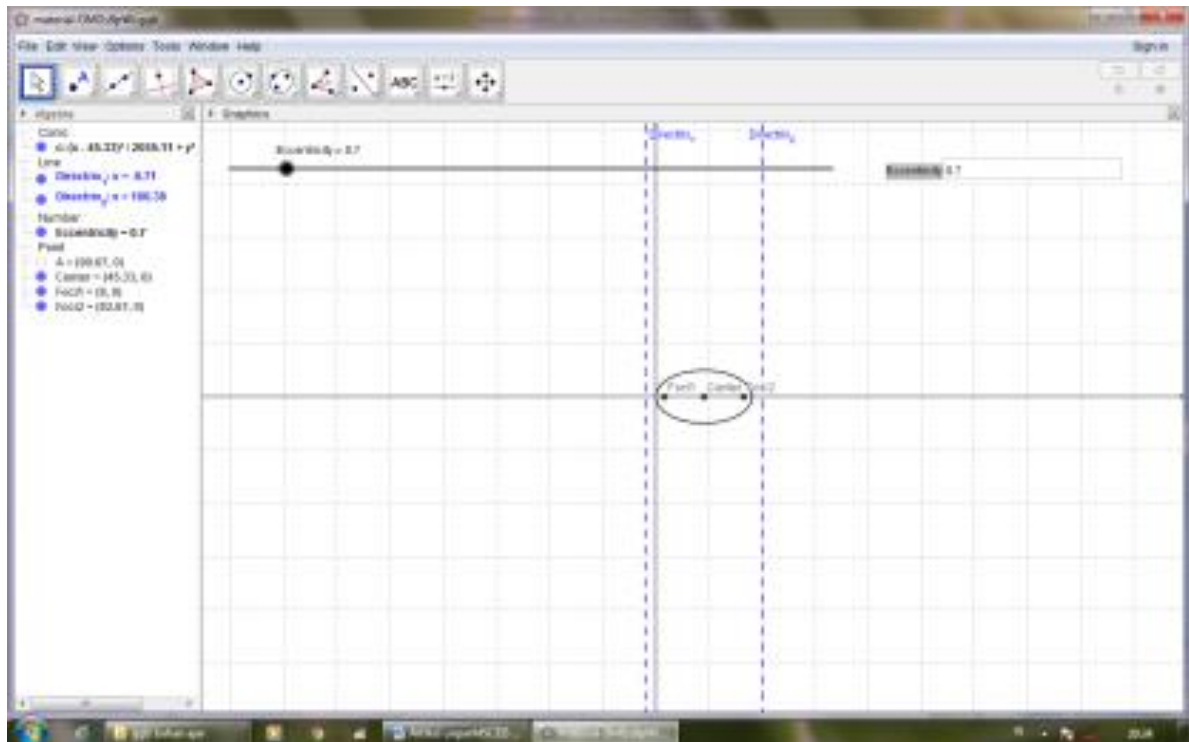


Figure 7 Task 4

On task 5 students sketched the Geogebra with  $e = 0.6$ . What images are obtained? Where are the boards and

directrix? The completion of this task is in Figure 8.

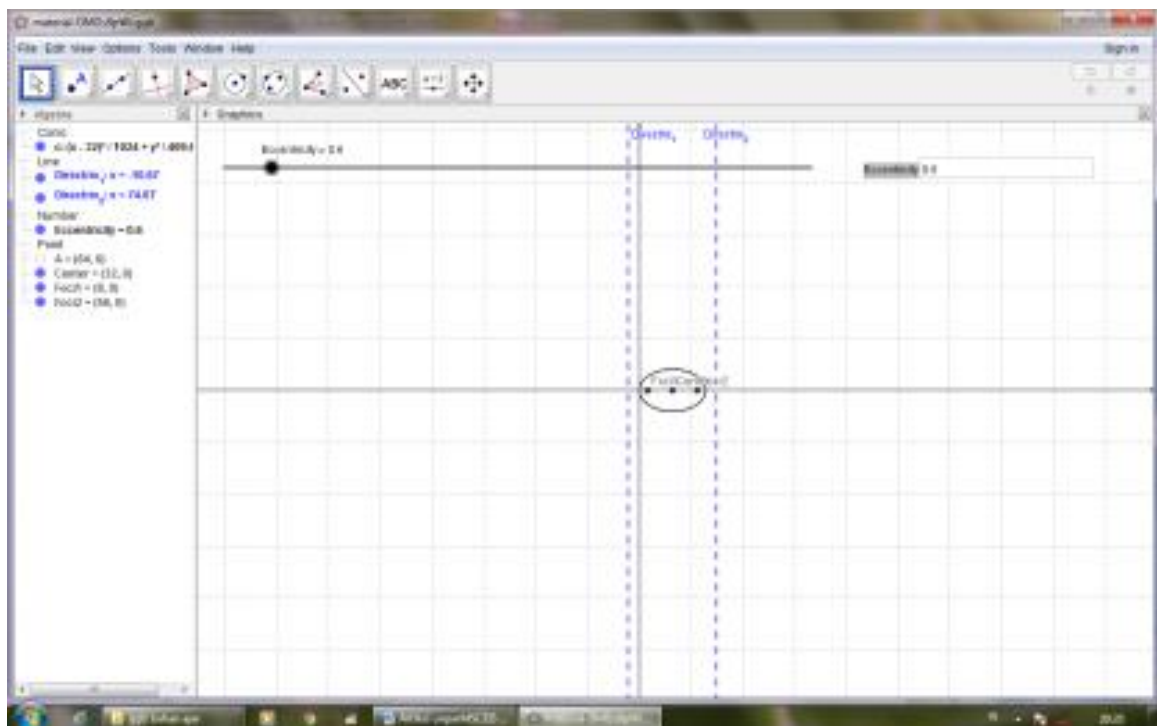


Figure 8 Task 5

On task 6 students sketched the Geogebra with  $e = 0.4$ . What images are obtained?

Where are the foci and the directrix? The completion of this task is in Figure 9.

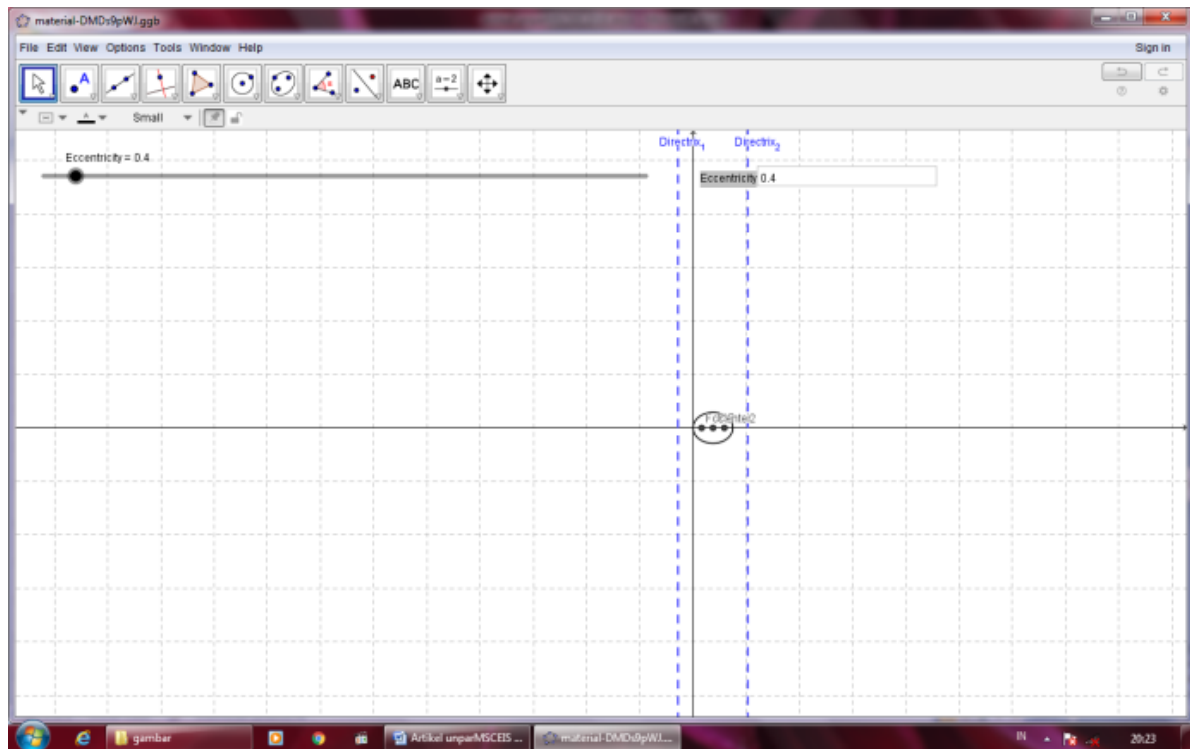


Figure 9 Task 6

On task 7 students sketched the Geogebra with  $e = 0$ . What images are obtained?

Where are the foci and the directrix? The completion of this task is in Figure 10.

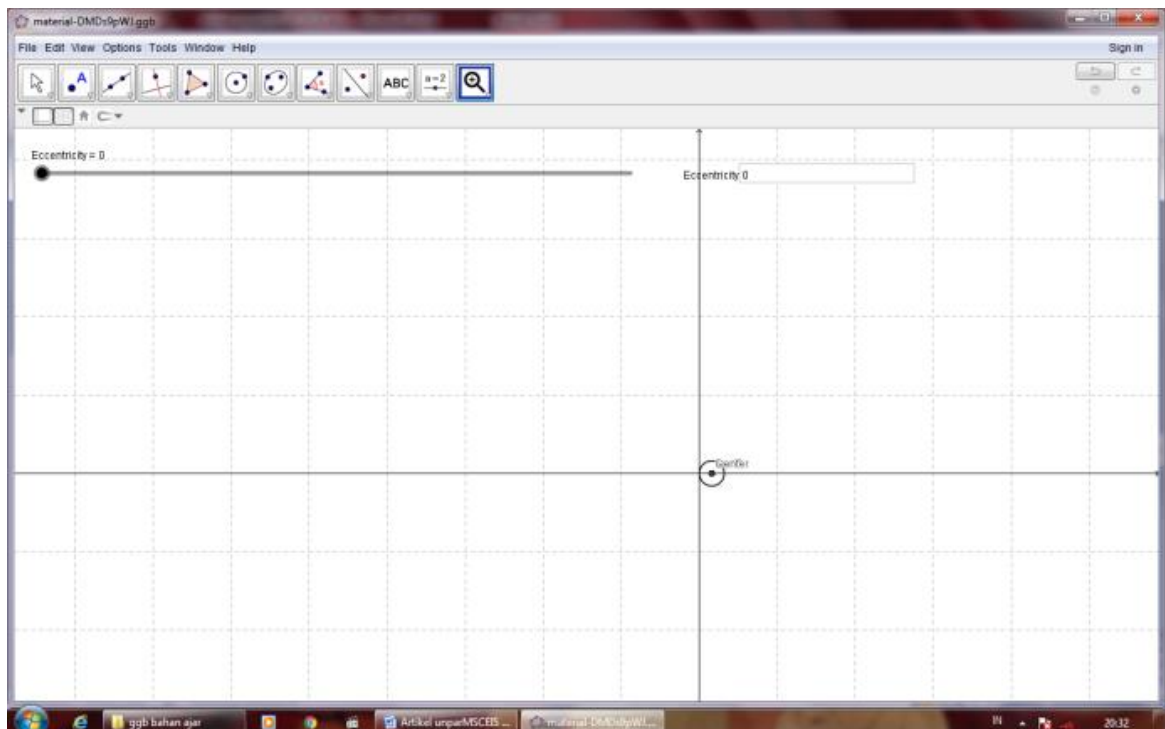


Figure 10 Task 7

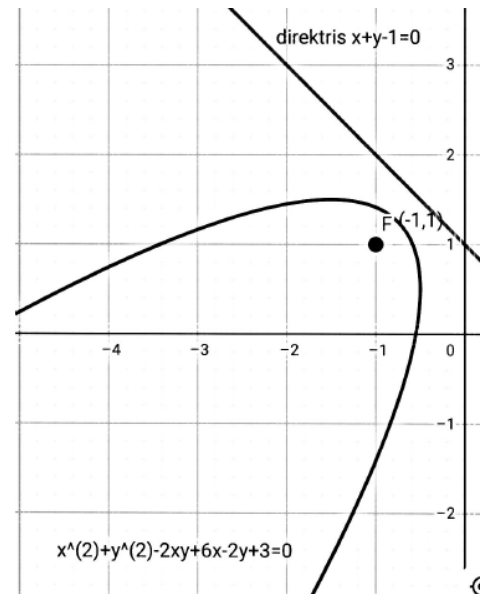
*Part III:* The task to conclude from the use of manipulatives and Geogebra

Based on the demonstration with manipulatives, on the ellipse A, B, and C it appears that the two foci of the ellipse are closer together. While the two directrix away each other. So for the ellipse D (circle), the two foci coincide to the center of the circle, whereas the two directrices are very far away. So the circle has eccentricity = 0. Through Geogebra has also seen that the two foci of the ellipse are closer to each other. While the two directrix away each other. So for the circle (the special ellipse), the two foci coincide into the center of the circle, while the two directrices are very far away. So the circle has eccentricity = 0.

Students are then given training. By using Geogebra students are asked to solve the following problem.

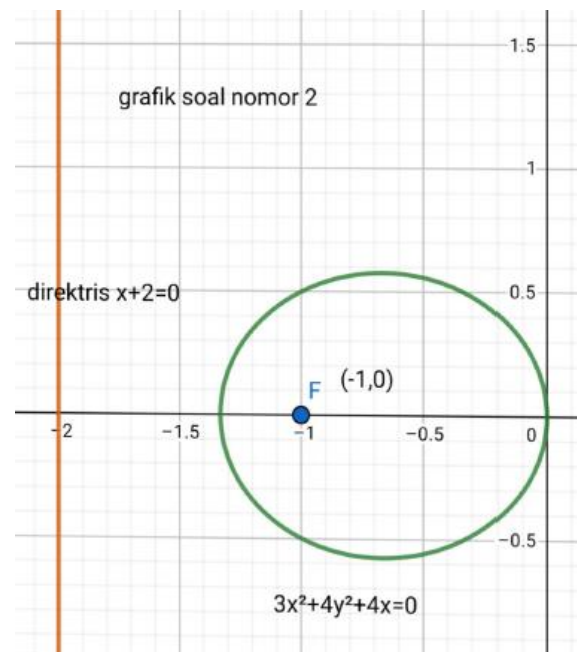
- 1) Find the equation and sketch the conic with Focus di  $(-1,1)$ , directrix  $x + y - 1 = 0$ , dan  $e = 1$ .
- 2) Find the equation and sketch the conic with Focus di  $(-1,0)$ , directrix  $x + 2 = 0$ , dan  $e = \frac{1}{2}$ .
- 3) Find the equation and sketch the conic with Focus di  $(1,2)$ , directrix  $3x - 4y + 1 = 0$ , dan  $e = \frac{5}{2}$ .

The problem was solved successfully by the students. Here are some student answers.



**Figure 11** The answer to student number 1

In question number 1 obtained the solution of the image sketch of a parabola, because of  $e=1$ .



**Figure 12** Student answer number 2

In problem number 2 obtained the solution of the drawing sketch of an ellipse, because of  $e<1$ .

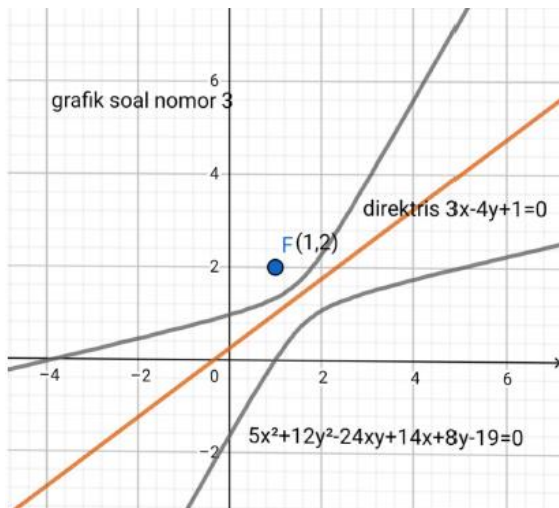


Figure 13 Student responses number 3

In question number 1 obtained the image sketch solution of hyperbola, because of  $e > 1$ .

Subsequently, the questions are solved algebraically using the conic definition.

1. Find the equation and sketch the conic with focus di  $(-1,1)$ , garis  $x+y-1=0$ , dan  $e=1$

Figure 14 Student answer number 1

In question number 1 is written  $|FP| = |DP|$  because  $e = 1$ . Then the equations are solved using algebra.

2. Find the equation and sketch the conic with focus di  $(-1,0)$ , garis direktris  $x+2=0$ , dan  $e = \frac{1}{2}$

$$\begin{aligned} \frac{|FP|}{|DP|} &= \frac{1}{2} \\ |FP| &= \frac{1}{2} |DP| \\ \sqrt{(x+1)^2 + (y-0)^2} &= \frac{1}{2} \left| \frac{x+2}{\sqrt{1^2 + 0^2}} \right| \\ (x+1)^2 + (y)^2 &= \frac{1}{4} \left( \frac{x+2}{1} \right)^2 \\ x^2 + 2x + 1 + y^2 &= \frac{1}{4} (x^2 + 4x + 4) \\ 4x^2 + 8x + 4 + 4y^2 &= x^2 + 4x + 4 \\ 3x^2 + 4y^2 + 4x &= 0 \end{aligned}$$

Figure 15 Student answer number 2

In question number 2 is written  $|FP| / |DP| = 1/2$ . Then the equations are solved using algebra.

3. Find the equation and sketch the conic with focus di  $(1,2)$  garis direktris  $3x-4y+1=0$ , dan  $e = \frac{5}{2}$

$$\begin{aligned} \frac{|FP|}{|DP|} &= \frac{5}{2} \\ |FP| &= \frac{5}{2} |DP| \\ \sqrt{(x-1)^2 + (y-2)^2} &= \frac{5}{2} \left| \frac{3x-4y+1}{\sqrt{3^2 + (-4)^2}} \right| \\ (x-1)^2 + (y-2)^2 &= \frac{25}{4} \left( \frac{3x-4y+1}{\sqrt{25}} \right)^2 \\ x^2 - 2x + 1 + y^2 - 4y + 4 &= \frac{25}{4} \left( \frac{9x^2 + 16y^2 - 24xy + 6x - 8y + 1}{25} \right) \\ 4x^2 + 4y^2 - 8x - 16y + 20 &= 9x^2 + 16y^2 - 24xy + 6x - 8y + 1 \\ 5x^2 + 12y^2 - 24xy + 14x + 8y - 19 &= 0 \end{aligned}$$

Figure 16 Student answer number 3

In question number 3 is written  $\frac{|FP|}{|DP|} = \frac{5}{2}$ . Then the equations are solved using algebra.

After doing the exercise the student concludes that the conic equation  $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$  with A, B, C, D, E, and F are real numbers with A, B, C not simultaneously equal to zero.

Based on the description, students complete the task well by using learning media. One of the alternative media used for mathematics learning to be meaningful is to utilize manipulatives. Mathematical manipulatives are concrete tools designed, created, and conceived intentionally used to help students understand concepts or principles in mathematics (Annisah, 2014). This circle manipulatives can be used as an alternative in learning. These manipulatives can serve as intermediaries for students to understand the abstract concept through the process of reconstructing knowledge through observation and experimentation. So that students experience learning meaningful and more motivated and enthusiastic in the learning process. Students are

basically learning from concrete things, while the concepts in abstract mathematics. So the media needed as intermediaries to understand abstract concepts. The types of media are media audio, visual media, audiovisual media, audiovisual silence, and audiovisual motion (Djamarah & Zain, 2010). The manipulatives we have designed are in the article (Purniati & Sudihartinih, 2015; Sudihartinih & Purniati, 2016, 2018) that are free to download from the internet. These manipulatives are certified Intellectual Property Rights in Indonesia. In addition to manipulatives, students can also use geometry software to facilitate their understanding.

Besides manipulatives, the success of students completing assignments is also assisted by Geogebra. Dynamic geometry software aims to enhance mathematics education (Chan & Leung, 2014). Geogebra is a software system that integrates the possibilities of both dynamic geometry and computer algebra in one tool for mathematics education. In teaching mathematics it might be used in many different ways, Geogebra for demonstration and visualization, a construction tool, discovering mathematics, and for preparing teaching materials (Hohenwarter & Fuchs, 2005). The advantage of computer manipulative pedagogical is 1) Providing another medium, one that can store and retrieve configurations, 2) Providing a manageable, clean, flexible manipulative, 3) Providing an extensible manipulative, 4) Recording and extending work (Clements, 1999)

## CONCLUSION AND IMPLICATION

Students complete the task well by using learning media. One of the alternative media used for mathematics learning to be meaningful is to utilize manipulatives

Besides manipulatives, the success of students completing assignments is also assisted by Geogebra. So that for the next learning can use manipulatives and Geogebra. Manipulatives and teaching materials that have been designed to be able to be submitted for copyright so that they can increase the amount of intellectual property in Indonesia.

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