

Involving Technology in The Teaching and Learning Mathematics

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Abstract

This study proposes a way to involve technology in the area of teaching mathematics. The technology tool is Geogebra⁴, which is computer software in the teaching and learning mathematics. This technology emphasizes on the use of multiple representations of mathematical concepts by computer software. The objective is to make students consider the representation of mathematical concepts and help them to enjoy studying mathematics. From that thought, hopefully student understanding will improve and their mathematical achievement will increase. The result of this study is five lesson plans for teaching and learning integral using Geogebra⁴.

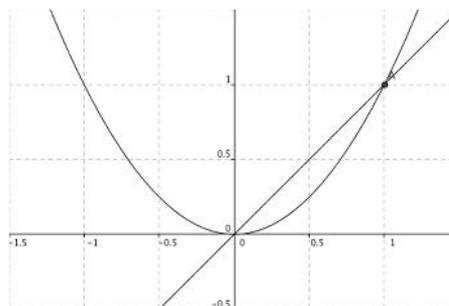
Keywords: *mathematics, education, technology, and geogebra⁴.*

PREFACE

Most teachers teach mathematics without representation. As a result, students have difficulties visualizing many concepts. The teachers just teach our students with the formula and symbol-letter, and then the students try to solve problems with the formula, without having knowledge of the visualization of the function or solution look likes. It is a little bit weird because students just remembering the formula without knowing what are the curves or representation look like.

For example, when teachers teach integral concept about area between two curves; teachers just give students the formula that is $\int_a^b \{f(x) - g(x)\}dx$, where $f(x)$ is above $g(x)$. For instance, teacher gives question about how to calculate the area between two functions,

$f(x) = x$ and $g(x) = x^2$, from $x = 0$ to $x = 1$. To solve this problem, the students in my country take minutes to draw picture of these two functions. For example, the picture is like this:



After drawing the picture, students begin to calculate the area, by solving the integral equation like this:

$$\int_0^1 x - x^2 \, dx = \left[\frac{1}{2}x^2 - \frac{1}{3}x^3 \right]_0^1 = \frac{1}{2} \cdot 1 - \frac{1}{3} \cdot 1 = \frac{1}{6}$$

Of course, for some students it is easy to draw the picture and to solve the integral equation. However, for the other students, it is hard to draw picture and to solve the integral, moreover when the functions are not simple like on the above example.

Technology (GeoGebra⁴) can help students get more understanding about this problem; that is how to make representation to calculate integral: area between two curves. My opinion is based on Bruner's insights on the role of representation have greatly influenced mathematics educator. Most of mathematics educators state that mathematical idea can be represented in three ways: enactively (concrete representation), iconically (pictorial representation), and symbolically (written symbols) (Bruner, 1960). In this context, technology (GeoGebra⁴) plays role as iconically or pictorial representation.

The purpose of this project is to involve technology in the mathematics classroom. I do believe that mathematics must be taught in a joyful learning environment with multiple representations, including representation from technology. With this project, I want to help

students to enjoy studying mathematics with technology, because nowadays, technology is the focus of their attention. I also want to make mathematics become less abstract with the representation from the technology (GeoGebra⁴). The result of this project is five lesson plans for teaching integral with GeoGebra⁴.

LITERATURE REVIEW

Most of the mathematics teachers just teach mathematics in the level of theory or concept. Students then have perception that mathematics is an abstract subject matter. The teachers rarely use representation or make connection between mathematics and the real life. From that historical background, I have a dream to change "the theoretical teaching style" in my country. In my teaching philosophy, I do believe that mathematics must be taught with realistic representation, involving technology, and trying to make students enjoy when they learn mathematics

My paper is based on Bruner's insights on the role of representation have greatly influenced mathematics educator. Most of mathematics educators state that mathematical idea can be represented in three ways: enactively (concrete representation), iconically (pictorial representation), and symbolically (written symbols) (Bruner, 1960). In this article, technology (GeoGebra⁴)

plays role as iconically or pictorial representation.

In the *Prepare and Inspire* (President's Council of Advisors on Science and Technology, 2010), there is a belief that technology has the potential to transform K-12 education, just as it has many other sectors of the US and global economy and our society. It can enable real-time and meaningful data gathering that allow learning and innovation in the education system. It can power innovative learning tools that prepare and inspire students. Furthermore, the report states that one of the most powerful tools to propel innovation in education is computation and information technology.

The report also explains that technology supports innovation in three fundamental ways: (1) continuous evaluation and improvement based on data, (2) rapid and inexpensive dissemination of successful solutions, and (3) mass customization. The report also notes some important points: (1) educational technology has been advancing rapidly in recent years and is likely to create major strides in the near future; (2) there will be a growing need for new instructional materials, new professional development materials, and new kinds of assessments that are aligned with higher standards and provide much richer learning experiences and more vibrant sources of information; (3) the "collection and use of data" is one of the U.S. Department of Education's four assurances; (4) technology is

becoming increasingly affordable, accessible, and versatile- a trend that will continue over the next decades, and will encompass personal and mobile devices; and (5) today's students are increasingly digital natives.

Many research findings conclude that technology is a great instrument to enhance mathematics teaching and learning process. Hatfield and Kieran (1972) explain that that was believe that "the activity of writing, processing, and studying the output of computer algorithms should promote the development of mathematical concepts and principles, computational skills, and problem-solving abilities of the students".

Ellington (2003) also states that when calculators were included in instruction, the ability to select the appropriate problem solving strategies improved for the participating student. Furthermore, she states that students who used calculators while learning mathematics reported more positive attitudes towards mathematics than their non-calculator using counterparts on survey taken at the end of calculator treatment.

From another research finding using computer-intensive algebra (CIA), O'Callaghan (1998) found that the CIA students achieved a better overall understanding of function and were better at the component of modeling, interpreting, and translating. Moreover, CIA students showed significant improvements in their attitudes toward mathematics,

were less anxious about mathematics, and rate their classes as more interesting.

From the other research findings, Kaput, Hegedus, and Lesh (2007) state that technology become infrastructure in mathematics education. They explain that technology is a fundamental yet invisible role similar to the electricity in our homes. Moreover, technology will lead to emphasize to new level and types of ideas and abilities, as well as new ways to think about traditional concepts and skills. In the school, technology will facilitate new type of social interaction and thinking, and new way to make mathematics less abstract and more accessible to a wider population of students. The authors said that to realize the potential of technology, new type of pedagogical diversification will be needed, and of course teacher development must be done.

Kaput et al. (2007) also show their result of classroom connectivity (CC) (i.e., classroom that involves technology); there are significant improvements in low-achieving students' abilities to solve standardize and applied problems. They also state that there are significant shifts in participation structures from non-CC to CC context. They also explain that the use of representationally rich software in mathematics education calls for a reconceptualization of both traditional and applied mathematics concepts. They also

see distinct differences in fundamental process such as posture and gesture as well as discourse, teachers using CC more positive and effectual in the classroom. They state that connectivity support pedagogical manipulation of students' focus of attention.

TECHNOLOGY OVERVIEW

Geogebra⁴ is an application for exploring and demonstrating Geometry and Algebra. It is an open source application and is freely available for non-commercial use. There are currently versions available for Windows, Mac OS X, Linux and other java-enabled platforms. To start Geogebra⁴ go to <http://www.geogebra.org> where we will see links to Web start or Download. The Web start option downloads the necessary java files to our computer and starts the application immediately. The advantage of choosing this option is that the application is always up to date. The Download option downloads files to our computer and we must then install. The big advantage here is that we can continue to work offline.

The installation process is very straightforward. After we have downloaded on a Windows machine just double-click the downloaded file. An Install Wizard will guide us through every step. It is strongly advised that we select the typical configuration when given the choice. Full instructions are given on the Geogebra4 site.

Double click the Geogebra⁴ icon on the desktop to start the application. We will be presented with a launch screen as shown in Figure 1.

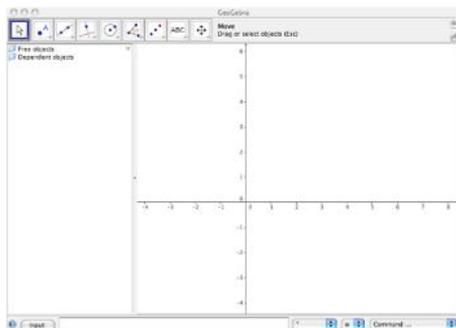


Figure 1: Launch Screen for GeoGebra⁴

The button menu along the top (see Figure 2) contains a submenu of actions. By clicking on the down-pointing arrow at the bottom right corner of any of these buttons the submenu is displayed.



Figure 2: Buttons for Geogebra⁴

LESSON PLANS OVERVIEW

On the next pages, five lesson plans about teaching integral using Geogebra⁴ will be explained. The first lesson plan is on teaching lower sum concept with representation from Geogebra⁴. In this lesson, students will investigate the properties of lower sum as a basic concept to understand Riemann integral. Students also will construct a conjecture and then they will try to

analyze their conjecture by Geogebra⁴.

The second lesson plan is teaching upper sum concept with representation from Geogebra⁴. In this lesson, students will investigate the properties of upper sum as a basic concept to understand Riemann integral. Students also will construct a conjecture about the relation between the number of rectangles and the value of the upper sum. Additionally, students will construct a conjecture and then they will try to analyze their conjecture by Geogebra⁴.

The third lesson plan is an investigation of the phenomenon when the number of rectangles goes to infinity. In this lesson, students will prove their conjecture in the previous lesson (the second lesson). They will construct the lower sum and the upper sum with a large enough number of rectangles, and then they will analyze whether their previous conjecture in the second lesson is true or not.

The fourth lesson plan is on teaching definite integral (Riemann Integral). In this lesson, students will investigate the properties of definite integral. Students also will construct a conjecture about the relation between the value of the definite integral (positive or negative) and the position of the area under the curves. Furthermore, students will construct a conjecture and then they will try to analyze their conjecture by

Geogebra⁴.

The fifth lesson plan is on teaching area between two curves with representation from Geogebra⁴. In this lesson, students will investigate the properties of area between two curves. Students also will construct a conjecture about the relation between the value of area between two curves and the position of the function f and g . Moreover, students will construct a conjecture and then they will try to analyze their conjecture by Geogebra⁴.

LESSON PLANS

Lesson Plan 1

Investigation: Lower Sum

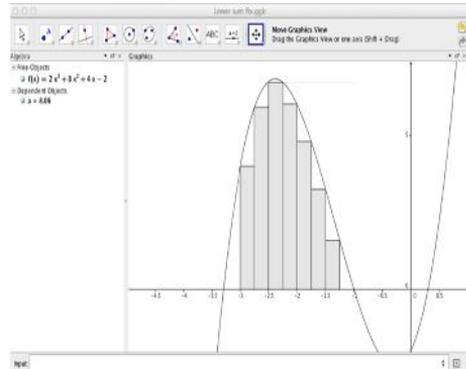
Lower Sum is $\sum_{k=1}^n f(m_k) \Delta x$, where m_k is x-value at which $f(x)$ attains a minimum on interval $[X_{k-1}, X_k]$. We can make Lower Sum representation in the Geogebra⁴ software.

Sketch

1. Type the equation $f(x) = 2x^3 + 8x^2 + 4x - 2$ in the Input Bar and press enter.
2. Type this command: Lower Sum [f, -3, -1, 8], or select it from the drop

down list in the Input Bar, and press enter.

You must have a representation like this:



Investigate

Use a slider to make an animation of the number of rectangles by clicking



the . You can set the number of rectangles between 8 and 100. What is the relationship between the number of rectangles and the Lower Sum value (a)? Analyze your findings to make a conjecture about the relationship between the number of rectangles and the Lower Sum values.

Conjecture: Write a conjecture below



value of the lower sum not change significantly.

Present Your Findings

Discuss your results with your partner or group. To present your findings you should:

1. Show some set of data about the relationship between the number of rectangles and the Lower Sum. For example, when the number of rectangles is _____, the lower sum value is _____.
2. From that data, you explain your own conclusion about the relationship between the number of rectangles and the Lower Sum values to your group members.

Explore More

See if you can come up with the number of rectangles that makes the

Investigation: Lower Sum (Teacher Comments)

Students Audience: High School

Prerequisite: Introduce (or let the student worksheet introduce) the terms Lower Sum

Geogebra⁴ Proficiency: Beginner

Class Time: 45 minutes

Construction Tips: This is a very simple construction that effectively illustrates the concept of Lower Sum

Sketch

Teacher have to makes sure that students have a representation as shown in the student worksheet. Teacher should moves to each group to see whether they have an intended representation.

Investigate/Conjecture

To increase the number of rectangles, create the slider n to go from 8 to 100 in steps of 0.1 by



clicking the . Then type this command: Lower Sum [f, -3, -1, n] or

select it from the drop down list in the Input Bar, and press enter.

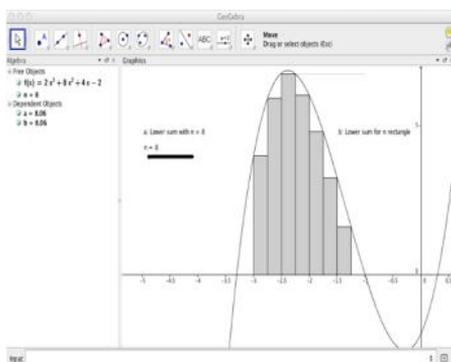
To make the difference between the value of Lower Sum clear, create text block

a: Lower Sum with n = 8 rectangles

by clicking  and create text block b: Lower Sum with n rectangles

by clicking .

The illustration is below:



Leading students to conjecture:

When the number of rectangles increases, then the value of Lower Sum also increases

Introduce the term infinity after students have made the conjecture. It is an important concept for

understanding Riemann integral in the next lesson.

Explore More

Lead students to a number of rectangles that makes the value of Lower Sum not change significantly. For example, n = 1000, this will illustrate the next lesson about the definition of the definite integral (Riemann Integral), which is the value of Lower Sum and the value of Upper Sum are equal when n goes to the infinity.

Lesson Plan 2

Investigation: Upper Sum

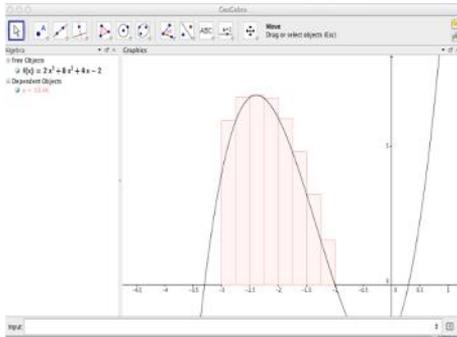
Upper Sum is $\sum_{k=1}^n f(M_k) \Delta x$, where M_k is x-value at which $f(x)$ attains a maximum on interval $[X_{k-1}, X_k]$. We can make Upper Sum representation in the Geogebra⁴ software.

Sketch

1. Type the equation $f(x) = 2x^3 + 8x^2 + 4x - 2$ in the Input Bar and press enter.
2. Type this command: Upper Sum [f, -3, -1, 8], or select it from the drop

down list in the Input Bar, and press enter.

You must have a representation like this:



Investigate

Use a slider to make an animation of the number of rectangles by clicking



the . You can set the number of rectangles is between 8 and 100. What is the relationship between the number of rectangles and the Upper Sum value (a)? Analyze your findings to make a conjecture about the relationship between the number of rectangles and the Upper Sum value.

Conjecture: Write a conjecture below

Present Your Findings

Discuss your results with your partner or group. To present your findings you should:

1. Show some set of data about the relationship between the number of rectangles and the Upper Sum. For example, when the number of rectangles is _____, the Upper Sum value is _____.
2. From that data, you explain your own conclusion about the relationship between the number of rectangles and the Upper Sum value to your group members.

Explore More

See if you can come up with the number of rectangles that makes the value of the upper sum not change significantly.

What do you think about the value of Lower Sum and the value of Upper Sum when n (the number of rectangles) becomes large enough? Make your conjecture here:

**Investigation: Upper Sum
 (Teacher Comments)**

Students Audience: High School

Prerequisite: Introduce (or let the student worksheet introduce) the terms Upper Sum

Geogebra⁴ Proficiency: Beginner

Class Time: 45 minutes

Construction Tips: This is a very simple construction that effectively illustrates the concept of Upper Sum

Sketch

Teacher have to makes sure that students have a representation as shown in the student worksheet. Teacher should moves to each group to see whether they have an intended representation, and help them if they have difficulties.

Investigate/Conjecture

To increase the number of rectangle, create the slider n to go from 8 to 100



in steps of 0.1 by clicking the . Then type this command: Upper Sum [f, -3, -1, n] or select it from the drop down list in the Input Bar, and press enter. To make the difference between the value of Upper Sum

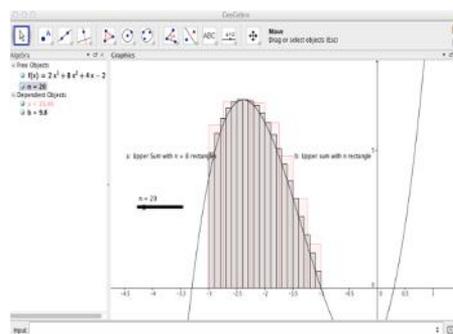
clear, create text block a: Upper Sum with $n = 8$ rectangles by clicking



and create text block b: Upper Sum with n rectangles by clicking



The illustration is below:



Leading students to conjecture:

When the number of rectangles increases, then the value of Upper Sum decreases

Teacher reminds students about the term infinity after students have made the conjecture. It is an important concept for understanding Riemann integral in the next lesson

Explore More

Lead students to a number of rectangles that makes the value of

Upper Sum not change significantly.
 For example, $n = 2000$.

Leading students' conjecture to the definition of definite integral (Riemann Integral), which is the value of Lower Sum and the value of Upper Sum are equal when n goes to the infinity. However, keep this definition for the next lesson.

Lesson Plan 3

Investigation: When the number of rectangles goes to infinity

You have made a conjecture about the relation between the value of Lower Sum and the value of Upper Sum when n (the number of rectangles) goes to infinity. Investigate your conjecture by following these procedures.

Sketch

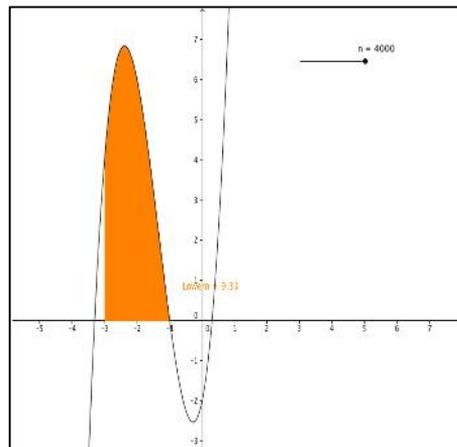
1. Type the equation $f(x) = 2x^3 + 8x^2 + 4x - 2$ in the Input Bar and press enter.
2. Type this command: Lower Sum [f, -3, -1, 8], or select it from the drop down list in the Input Bar and press enter.
3. Use a slider to make an animation of the number of rectangles by



clicking the , moving n between 8 and 4000.

4. Type this command: Lower Sum [f, -3, -1, n] or select it from the drop down list in the Input Bar and press enter.

You will see this representation:



Investigate

Use the same procedures as above to create a representation for Upper Sum. You can set the number of rectangles between 8 and 3000, and call it m . What do you find about the relation between the value of Lower Sum and the value of Upper Sum when n and m (the number of rectangles) go to infinity? How about your previous conjecture, is it true? Write your findings on the space below.

Investigation: When the number of rectangles goes to infinity (Teacher Comments)

Students Audience: High School

Prerequisite: Introduce (or let the student worksheet introduce) the terms Upper Sum

Geogebra⁴ Proficiency: Beginner

Class Time: 45 minutes

Construction Tips: This is a very simple construction that effectively illustrates the concept of Lower Sum and Upper Sum when the number of rectangles goes to infinity.

Sketch

Teacher have to makes sure that students have a representation as shown in the student worksheet. Teacher should moves to each group to see whether they have an intended

representation, and help them if they have difficulties.

Investigate/Conjecture

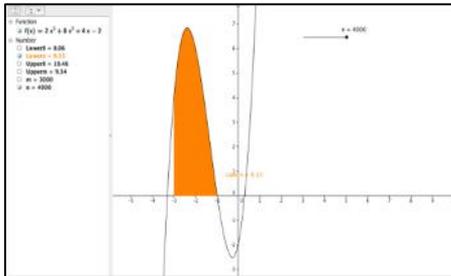
As students construct their Upper Sum representation, make sure that they do these procedures:

1. Type the equation $f(x) = 2x^3 + 8x^2 + 4x - 2$ in the Input Bar and press enter
2. Type this command: Upper Sum [f, -3, -1, 8] or select it from the drop down list in the Input Bar and press enter.
3. Use a slider to make an animation of the number of rectangles (m) by clicking the , moving m between 8 and 4000.
4. Type this command: Upper Sum [f, -3, -1, n] or select it from the drop down list in the Input Bar, and press enter.

Also make sure that students give different names for each variable in the left hand side of the Gogebra⁴ interface. For example, for the Lower Sum with 8 rectangles, students can call it Lower 8, and for the Lower Sum with n rectangles, students can call it Lower n, and likewise for the

Upper Sum. So, students can look at those different variables easily.

The illustration is below:



Lead the students to make final conclusion about their conjecture, that is the definition of definite integral (Riemann Integral), which is the value of Lower Sum and the value of Upper Sum are equal when n and m (the number of rectangles) go to the infinity.

Lesson Plan 4

Investigation: Definite Integral

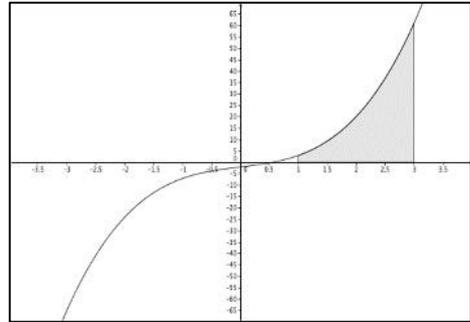
A definite integral is an integral $\int_b^a f(x)dx$ with upper and lower limits. If x is restricted to lie on the real line, the definite integral is known as a Riemann Integral. We can make definite integral representation in the Geogebra⁴ software.

Sketch

1. Type the equation $f(x) = 2x^3 + 3x - 2$ in the Input Bar and press enter.

2. Type this command: Integral [f, 1, 3], or select it from the drop down list in the Input Bar, and press enter.

You must have a representation like this:



Investigate

Investigate the definite integral from the same function $f(x) = 2x^3 + 3x - 2$ between $x = -3$ and $x = -1$. What is the result? Do you have a positive or a negative number of results? What your conclusion. Write your conclusion as a conjecture.

Conjecture: Write a conjecture below

Present Your Findings

Discuss your results with your partner or group. To present your findings you could:

1. Show some set of data about the differences between definite integral that has area above x -axis and definite integral that has area below x -axis. For example, when the upper and lower limits are 1 and 3 (above x -axis), the definite integral is _____, and when the upper and lower limits are -3 and -1 (above x -axis), the definite integral is _____.
2. From that data, then you explain your conjecture about the definite integral that has an area above x -axis and the definite integral that has an area below x -axis to your group members.

Explore More

See if you can come up with a conclusion why the definite integral that has area under x -axis is negative?

Write your thought here:

Investigation: Definite Integral (Teacher Comments)

Students Audience: High School

Prerequisite: Introduce (or let the student worksheet introduce) the terms Definite Integral

Geogebra⁴ Proficiency: Beginner

Class Time: 45 minutes

Construction Tips: This is a very simple construction that effectively illustrates the concept of Definite Integral (Riemann Integral)

Sketch

Teacher have to makes sure that students have a representation as shown in the student worksheet. Teacher should moves to each group to see whether they have an intended representation, and help them if they have difficulties.

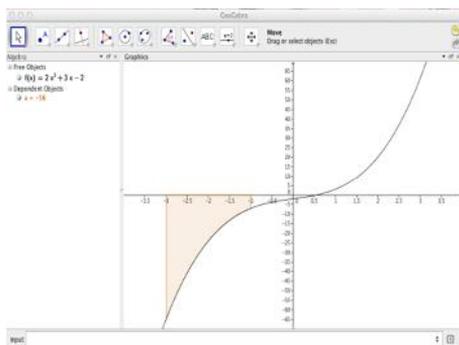
Investigate/Conjecture

To make a representation of the definite integral that has upper and

lower limit -3 and -1, you can follow this procedure:

1. Type the equation $f(x) = 2x^3 + 3x - 2$ in the Input Bar and press enter.
2. Type this command: Integral [f, -3, -1] or select it from the drop down list in the Input Bar, and press enter.

The illustration is below:



Leading students to the conjecture:

Definite integral that has area above x-axis is positive and definite integral that has area below x-axis is negative

Teacher should use Geogebra⁴ to show some example of representations to make sense this conjecture.

Explore More

Leading students to the conclusion that: the definite integral that has

area below x -axis is negative because $f(x) \leq 0$ in the whole interval $[-3, -1]$. Remember the definition of the Riemann Sum: $\sum_{k=1}^n f(x_i) \Delta x$ as the basic concept of the definite integral. Therefore, if $f(x_i) \leq 0$ for all $x_i \in [-3, -1]$ then Riemann Sum is a negative number.

Lesson Plan 5

Investigation: Area Between Two Curves

The method for determining the area between two curves is an important application of integral calculus. It lets us determine the area of non-standard shapes by evaluating the definite integral. You will learn and investigate that in this lesson.

Sketch

1. Type the equation $f(x) = x^2$ in the Input Bar and press enter.
2. Type the equation $g(x) = 2x + 1$ in the Input Bar and press enter.

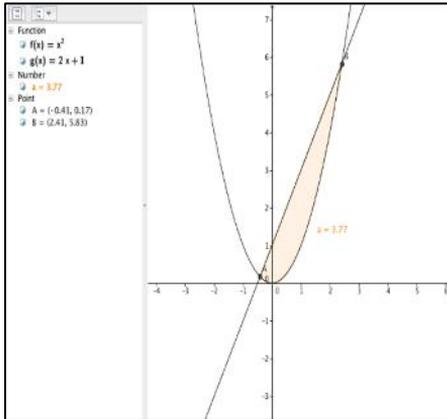


3. Click on f and g to create the points of intersection, A and B.
4. Create the number a , which is the area between the line $g(x)$ and the

curve $f(x)$ by typing in the Input Bar: $\text{Integral}[g(x), f(x), (A), x(B)]$

plane and the result of the area between f and g .

You must have a representation like this:



Conjecture: Write a conjecture below

Present Your Findings

Discuss your results with your partner or group. To present your findings you could:

Investigate

Construct the same procedures as above, but for step 4, you should type: $\text{Integral}[f(x), g(x), (A), x(B)]$. What happens with the result of the area; is it positive or negative? Write your answer here

You know that the area of any shape must be a positive number. If you find a negative number for the representation above, why does it happen? Analyze your answer, and form a conjecture that explains the relation between the position of function f and g in the coordinate

1. Show some set of data about the relation between the area of function $f(f(x) = x^2)$ and $g(g(x) = mx + c)$. For example, when you type f first, and then you type g , the area is _____, and when you type g first, and then you type f , the area is _____.

(Note: you can change the number of m and c in the function



you like with the slider)

2. From that data, then you explain your own conclusion about the relation between the position of function f and g and the value of

the area (between the two functions) to your group members.

Investigation: Area Between Two Curves (Teacher Comments)

Students Audience: High School

Prerequisite: Introduce (or let the student worksheet introduce) the terms Area Between Two Curves

Geogebra⁴ Proficiency: Beginner

Class Time: 45 minutes

Construction Tips: This is a very simple construction that effectively illustrates the concept of Area Between Two Curves

Sketch

Teacher have to makes sure that students have a representation as shown in the student worksheet. Teacher should moves to each group to see whether they have an intended representation, and help them if they have difficulties.

Investigate/Conjecture

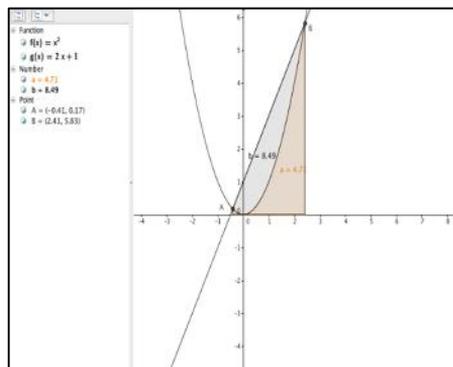
As students manipulate their integral: area between two curves representation, you should note that if we type: Integral [f(x), g(x), x(A), x(B)], the area must be a negative number. The reason is the area under f function is less than the area

under g function on that interval. So, if you type: Integral [f(x), g(x), x(A) , x(B)], it means you subtract the area under function g from the area under function f, and it must be a negative number.

To make this representation clear, you can do these procedures and show it to the students:

1. Create the number a , which is the area between the line $g(x)$ and the x -axis by typing in the Input Bar: Integral[g(x), x(A) , x(B)]
2. Create the number b , which is the area between the curve $f(x)$ and the x -axis by typing in the Input Bar: Integral[f(x), x(A) , x(B)]

This is the illustration:

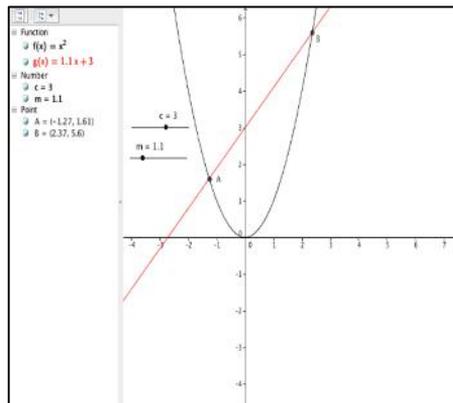


This representation shows that the area under g, which is $b = 8.49$, and the area under f, which is $a = 4.71$. Therefore, if you subtract the area

under function g from the area under function f , the result must be -3.78 .

Leading students to the conjecture:

When the position of g is above f in the coordinate plane, we must type g first, and then type f in the Input Bar (it means we subtract the area under function f from the area under function g), in order to get a positive number for the area between two curves.



Present Your Findings

When students present their findings in a group discussion, you could guide them when they construct general linear function $g(g(x) = mx + c)$ by these procedure:

1. Type the equation $f(x) = x^2$ in the Input Bar, and press enter.
2. Create the slider c to go from 0 to 5 in steps of 0.1.
3. Create the slider m to go from 0 to 5 in steps of 0.1.
4. Type the equation $g(x) = mx + c$ in the Input Bar, and press enter.
5. You will get this representation:

SUMMARY

There are many factors that have implications in the teaching and learning mathematics. Teachers must consider any changes around the world in order to make more adaptable mathematics teaching and learning processes. A variety of technological tools have been produced and it has a big impact in our society. In order to adapt with the new era of connectivity, teachers have to learn that technology. Geogebra⁴ emphasizes on the use of multiple representations of mathematical concepts by computer software. The objective is to make students consider the representation of mathematical concepts and help them to enjoy studying mathematics. From that thought, hopefully student understanding will improve and their mathematical achievement will increase.

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