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The Importance of Mathematical Representation Ability for Elementary School Students: A Literature Review and Its Implications

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Abstract: The purpose of this article is to discuss the importance of mathematical representation ability for students in solving various mathematical problems. To achieve this objective, a literature review method is employed, describing the mathematical representation ability based on the opinions of several experts and various previous studies with supporting reference. The development of mathematical representation ability is highly crucial for elementary school students, as it forms the foundation for understanding mathematical concepts at higher levels. When students posses and master mathematical representation abilities from an early stage, they will find it easier to tackle problems related to different mathematical topics and materials at more advanced levels. Generally, there are three types of mathematical representation abilities that can be developed, namely verbal representation, visual representation, and symbolic representation . The analysis of these mathematical representation abilities can serve as a reference in planning research related to the enhancement of students' mathematical representation abilities.

Keywords: mathematical representation ability, verbal representation, visual representation, symbolic representation

INTRODUCTION

The mathematical abilities demanded by NCTM (2000) consist of: mathematical problem solving , mathematical reasoning , mathematical communication, mathematical connections, and mathematical representation. These five mathematical abilities are crucial to be mastered and developed by students in learning mathematics. This is because these five abilities are highly relevant to the abilities that students must possess in facing the competition in the twenty-first century.

One of the important mathematical abilities that students need to possess and develop is mathematical representation ability. Mathematical representation ability serves as a bridge that connects students to understand other mathematical abilities. This means that when students have mastered and can develop mathematical representation ability effectively, they will easily grasp other mathematical abilities. In other words, mathematical representation ability is the key to mastering other mathematical abilities.

However, the importance of this mathematical representation ability does not align with the existing reality. The level of mathematical representation ability among students in

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Indonesia, especially at the elementary education level, is still relatively low. This is based on research finding conducted by Legi (2008), Ani Minarni, et al (2016) and Sulastri, et al (2017) which revealed that students' achievements in understanding mathematics and representation tests are categorized as low. The low level of mathematical representation ability is caused by various factors. Among them, students still struggle to comprehend how to represent real-world problems into representative mathematical problems (Suryowati (2015) and Sulastri (2017)). Additionally, students are not adequately trained in creating various representations as they have mainly focused on solving problems using given formulas (Hudiono (2005) and Devi Aryanti, et al (2013)). Furthermore, Bossé, et. al. (2016) also mentioned that some teaching methods involving representations may confuse students, leading to a lack of clear understanding of mathematical concepts.

Based on various issues concerning the mathematical representation ability, solutions are required to address them. One of the solutions is to recommend a model or form of teaching that can support the development of students' mathematical representation abilities. Therefore, at the end of this study, the authors will recommend one effective form of teaching approach that can enhance and improve the mathematical representation ability of elementary schools students.

METHODS

This article was written using a literature review method on the topic at hand. According to Nazir (1988), literature review is a data collection technique that involves reviewing various books, literature, notes, and reports related to the problem that needs to be adressed. Meanwhile, according to Sarwono (2006), literature review is the study of various reference books and previous research results that are useful to establish the theoretical foundation for the problem to be investigated. Based on the understanding of the experts mentioned above, this article will discuss the mathematical representation ability obtained from several supporting sources on the topic.

RESULT AND DISCUSSION

1. Mathematical Representation Ability

NCTM (2000) reveals that representation is a way that someone uses to communicate answers or mathematical ideas in question. Representation is the translation of a problem or idea in a new form, including pictures or physical models into symbols, words or sentences.

Furthermore, it is also disclosed that representation refers to external objects or manifestations (for example, concrete; pictorial, diagrammatic, or graphic; algebraic or symbolic; verbal, spoken or written; and numerical or tabular) that can be used to represent and transform mathematical ideas. (Duval, 2006; Goldin, 1998) in Bosse, et. al (2016). Mathematical representations are also defined as symbolic schemes articulated through combinations of characters and syntactic rules that define methods of depicting and transforming mathematical relationships (Bossé, Adu-Gyamfi, & Chandler, 2014; Cobb, Yackel & Wood, 1992; Duval, 1999, 2006 ; Goldin, 1987; Kaput, 1987a; 1987b; Sims-Knight & Kaput, 1983) in Bosse, et. al (2019) .

Mathematical representations encode and communicate mathematical ideas, relate mathematical ideas to a larger field of reference, and provide mechanisms through which

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ideas can be further investigated and modified. In particular, different representations use different nomenclature to communicate ideas (Bosse, et, al, 2019) . Salkind (2007) mentions that representation is a configuration. In general, a representation is a configuration that can express something else in some way. Hwang, *et. al* (2007) states that in general psychology, representation means the process of making concrete models in the real world into abstract concepts or symbols. Pape and Tchoshanov (2001) state that representation is an internal abstraction of mathematical ideas or cognitive schemes that students construct through experience. Mustangin (2015) states that representation is a model or substitute for a problem situation or an aspect of a problem situation that is used to find a solution.

From the several definitions that have been put forward, it can be concluded that the ability of mathematical representation is the ability to express a mathematical idea that is displayed as a form that represents a problem situation in the form of an external object or embodiment (for example, concrete; pictorial, diagram or graphic; algebraic or symbolic; verbal, verbal or written; and numerical or tabular) to find a solution to the problem.

2. The Role of Representational Ability in Learning Mathematics

Representation plays a very important role in efforts to develop and optimize students' mathematical abilities. NCTM in *the Principles and Standards for School Mathematics (Standars, 2000)* includes representation *as* the fifth process standard after *problem solving, reasoning, communication, and connection* . According to Jones (2000) in Sabirin (2016) several important underlying reasons are as follows: (1). Fluency in translating between different forms of representation is a fundamental ability that students need to have in order to build concepts and think mathematically; (2). The teacher's way of presenting mathematical ideas through various representations will have a huge influence on students' understanding of learning mathematics; and (3). Students need practice in building their own representations so that they have strong and flexible conceptual skills and understanding that can be used in solving problems. In learning, through external representations of students, the teacher can guess what is actually happening which is an internal representation in the minds of students, so that the teacher can take the right steps to bring students to learn.

Studies on the importance of mathematical representation in problem solving were carried out by Bal (2014), Caglayan & Olive (2010), and Villegas, Castro, & Gutiérrez (2009). These studies found that students' representational ability was a key aspect for success in solving math problems and understanding math concepts. According to Castro-Rodrigues, Pitta-Pantazi, Rico and Pedro (2016), representation is considered as a tool in the process of forming concept meaning, which is closely related to students' conceptual knowledge (Son & Lee, 2016). Mathematics requires representation because it is abstract (Minarni & Napitupulu, 2017). Students who are able to represent mathematical ideas correctly have a good understanding of mathematical concepts. Mathematical representation is an important skill in learning mathematics that allows students to interpret and solve problems easily (Supandi, et al, 2018).

In education, mathematical representations can help make abstract mathematical concepts accessible to students (Goldin, 2002) and enable two-way student-teacher communication of mathematical ideas and understanding (Lesh, Post, & Behr, 1987). Various studies acknowledge the centrality of representation in the teaching and learning of mathematics (eg, Brenner, et al, 1997; Brenner, Herman, Ho, & Zimmer, 1999; Knuth, 2000) and many note the gains in learning through multiple representations (eg, Acevedo Nistal, et al, 2009; Ainsworth, 2006; Rau, 2016). Some equate learning mathematics with exploring the representations and relationships among ideas presented (McKendree, Small, Stenning, & Conlon, 2002).

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Based on some of the descriptions above, researchers can draw conclusions that the ability to represent mathematics is one of the most important skills and must be possessed by students in order to understand concepts in mathematics. By having the ability to represent mathematics, students are able to identify problems and analyze the interrelationships between concepts, which in turn are able to solve mathematical problems in their own way, so that understanding of these concepts can be more easily achieved.

3. Types of Mathematical Representational Ability

Representation is a process and a product (NCTM, 2000). As a product, representation refers to forms of external representation (Goldin, 1998) such as symbols, graphs and diagrams. As a process, it is seen as an internal thought in the minds of teachers and students when working with representations. Representation can then be viewed as a useful means of communicating mathematical ideas. More specifically, students demonstrate their ability to connect mathematical ideas when they are able to translate between different representations of concepts fluently, resulting in a deeper and meaningful understanding of mathematics (NCTM, 2000).

According to Goldin and Kaput (2015), also revealed that mathematical representation is divided into two parts, namely internal representation and external representation. Internal representation leads to individual mental configurations both for students and for *problem solvers.* Because internal representations are mental configurations, of course they cannot be observed directly. As a teacher or researcher can regularly infer students' mental configurations through what students do and what they say. What students do and say is called external representation. Often teachers/researchers make such conclusions tacitly and not explicitly, and sometimes teachers consciously begin to develop a certain kind of internal representation of students through teaching activities.

Mustangin (2015) states that the various representations that are often used in communicating mathematics include: (1) visual presentations such as tables, pictures, graphs; (2) mathematical statements or mathematical notation; (3) written texts written in their own language, both formal and informal, or a combination of all. The same thing was expressed by Neria and Amit (2004) who stated that the various representations that are often used in communicating mathematics include: tables, pictures, graphs, mathematical statements, written text, or a combination of all of them. Kartini (2009) states that basically representations can be classified into (1) visual representations (pictures, graphic charts, or tables), (2) symbolic representations (mathematical statements/mathematical notation, numeric/algebraic symbols) and (3) representations verbal (written text / words).

The use of all types of representations can be made complete and integrated in testing the same problem or in other words, mathematical representations can be made in various ways (multiple representations). Lesh, Post and Behr (in Hwang, Chen, Dung, & Yang, 2007) divide the representations used in mathematics education into five types, including representations of real world objects, concrete representations, representations of arithmetic symbols, representations of spoken or verbal language and representations of images or graphics.

4. Mathematical Representation Ability Indicator

As one of the process standards, NCTM (2000) sets representation standards that are expected to be mastered by students during learning at school, namely:

a. Create and use representations to recognize, record or record, and communicate mathematical ideas;

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b. Selecting , applying, and translating between mathematical representations to solve problems;
c. Using representations to model and interpret physical, social, and mathematical phenomena.

Ability Students' mathematical representation can be measured through several indicators of mathematical representation ability. Indicators of students' mathematical representation according to Santia, et al (2019) are presented in the following table:

Tabel. 1 Mathematical Representation Ability Indicator

Types of Mathematical Representations	Description	Mathematical Representation Indicator
Verbal Representation	State problems, problem solving strategies, and monitor results and discussions orally.	• The problems presented are similar to text problems. • Problems expressed, change a few words with his own style of speech. • State the problem by writing the same text as the problem. • State the problem by writing a paraphrase of the question. • Describe the strategy developed to obtain a solution. • Describes the strategies developed to find solutions and no solutions to paper problems • Explain the arguments and facts in support of the chosen solution orally. • Explain the arguments and facts that support the chosen solution by writing text. • Describes the evaluation results and the effectiveness of suitable solutions orally.
Visual Representation	State problems, problem-solving strategies, and monitor results and discussions using embodiments of pictures, graphs, diagrams, number lines, and other mathematical images.	• State problems by creating pictures, graphs, diagrams, number lines, and other mathematical drawings on worksheets • Describes strategies developed for obtaining solutions and alternative solutions to problems with pictures, graphs, diagrams, number lines, and other mathematical images on worksheets. • Describes the arguments and facts in favor of the chosen solution by pictures, graphs, diagrams, number lines, and other mathematical drawings on worksheets. • Explaining the results of the evaluation and the effectiveness of solutions expressed by pictures, graphs, diagrams, number lines, and other mathematical images on worksheets.
Symbolic Representation	State problems, problem-solving strategies, and monitor results and discussions using numbers, variables and other	• Express problems using numbers, variables, and other mathematical expressions on worksheets. • Describes the strategies developed for obtaining solutions and alternative solutions to problems using numbers, variables, and other mathematical expressions in a worksheet. • Explain the arguments and facts that support the chosen solution using numbers, variables, and other

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	mathematical expressions.	mathematical expressions on the worksheet. • Explaining the results of the evaluation and the effectiveness of solutions expressed using numbers, variables, and other mathematical expressions on worksheets.

Based on these indicators, the teacher can determine what steps should be taken to develop students' mathematical representation abilities . Either by choosing the right learning strategy, or the media that can be used to support the strategy.

Various Previous studies have reported developments in the ability to represent mathematical s . Among them is research conducted by Cia Su Ngin (2018) which describes the use of multiple representations in various mathematical discourses by using mediation flowcharts to describe and analyze mathematical representations made by teachers in percent problems. This research has provided some important insights into the complexities of mathematical communication through multiple representations. So the researchers suggest that this analysis with a cognitive perspective be carried out for other materials in mathematics.

The research conducted by Delsika Pramata Sari, et al (2018) analyzed the errors experienced by students who studied with REACT strategies and traditional learning in solving mathematical representation abilities problems. From the results of his research, it can be concluded that the most prominent error in the ability of mathematical representation is in the indicator of solving problems with arithmetic symbols (symbolic representation). The error is about the concept of the surface area of a prism involving symbols that need to solve problems by applying the Pythagorean concepts, the area of a triangle, and the area of a square. In addition, many students experience errors with traditional learning on indicators of making pictures of real-world situations to clarify problems and facilitate their resolution (visual representation), namely the concept of the surface area of a cube with contextual problems.

Furthermore, related to learning strategies that can support the improvement of mathematical representation abilities have also been recommended by several previous studies. Among them research conducted by Ani Minarni & E. Elvis Napitupulu (2017) concluded that *Joyful Problem Based Learning (JPBL)* is a learning strategy that can be applied to improve *Mathematical Representation Ability (MRA)* . The research results of Supandi, et al (2018) show that students who use the *think-talk-write strategy* have greater abilities in mathematical representation than those who use the expository strategy. The results of this study also show that the level of *self-efficacy* significantly affects students' mathematical representation abilities. In addition, I Made Ari Purwadi, et al. (2019) also conducted research which resulted in findings that *the Concrete, Pictorial, Abstract (CPA)* strategy had significantly influenced students *' Mathematical Conceptual Understanding* (*MCU*) and students *' Mathematical Representation (MR)* in fractions. The CPA strategy improves student MCU and student MR where it gives students (1) the ability to explore and experience fractions, (2) to be more motivated and excited when learning, (3) to be more enthusiastic when expressing ideas in own words and own reasons, (4) to better communicate their findings, (5) to get better results in answering fraction problems. This learning strategy significantly improves students' ability to meaningfully solve problems involving fractions.

Representational ability Mathematics is one of the abilities that is not static, but can be developed and improved. In order to develop mathematical representation skills, an appropriate method is needed to obtain optimal results. According to Michael J. Bossé, et. al (2016) required the use of appropriate concrete representations in building students' understanding of mathematical concepts. In addition, learning models are also needed that

support the development of mathematical representation abilities. Sulastri, et al (2017) revealed that teachers need to choose and use the right learning approach, so that the learning process takes place optimally and is able to develop mathematical representation abilities.

One alternative form of learning that the authors recommend is a learning approach that can effectively develop and improve the mathematical representation abilities of elementary school students, namely learning with a visualization approach. This learning approach developed by Joan Cohen Jones (2012) uses different types of visual representations to describe each mathematical concept. This lesson presents images that are new and always different in each lesson, but are images that are close to students' lives. These ever-new images are expected to provide unique opportunities for learning. Specifically, this visualization approach offers elementary school teachers the opportunity to learn about mathematics and teach mathematics in the real world. For example, by using classrooms, photos and illustrations, technology, video clips, multicultural perspectives, and children's literature. This approach is successful in capturing the attention of prospective teachers, helping them understand the relevance of mathematics to life and giving them tools for teaching in the 21st century.

CONCLUSIONS

From the description above, it can be concluded that the ability of mathematical representation is very important to be mastered and developed by students, especially at the Basic Education level. By mastering this mathematical representation ability, it is expected that students will find it easier to master various other mathematical abilities. In general, there are three types of mathematical representation abilities that can be developed, namely verbal representation, visual representation, and symbolic representation. These three types of representation have indicators that can be used as a reference in developing them. The form of learning that is recommended to be able to develop students' mathematical representation abilities is one of them is the visualization approach. By using this approach, it is hoped that students will be better able to achieve the various indicators that have been set.

The results of this analysis regarding the ability of mathematical representation can be used as a reference in planning research related to the development of students' mathematical representation abilities . In addition, it is hoped that the recommended forms of learning can be implemented in a comprehensive manner so that it can be more optimal to see the achievement of various indicators of students' mathematical representation abilities.

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