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Exploring Grade 8 Students' Procedural Skills in Solving Pythagorean Problems

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This study examines the procedural skills of eighth-grade students in solving Pythagorean Theorem problems, focusing on the patterns and variety of steps they use. A qualitative case study approach was employed, involving three students with varying levels of mathematical ability: high, medium, and low. Data were collected through problem-solving tasks and semi-structured interviews. The results showed that students with high mathematical ability demonstrated structured and accurate procedural skills, making no errors in their solutions. In contrast, students with medium ability displayed inconsistent procedural skills, often failing to record key information and not reflecting on their answers. Students with low mathematical ability frequently skipped structured steps and jumped directly to solutions without considering the process. These findings highlight the importance of instructional approaches tailored to students' skill levels, such as structured learning and scaffolding, to strengthen procedural skills. The results of this research can inform the design of a mathematics learning module that accommodates varying levels of student ability. This study also suggests that further research should expand the sample size to validate these findings and explore learning strategies that can enhance students' procedural skills through scaffolding and guided practice.

K e y w o r d s :
Procedural skills; geometry; Pythagoras; mathematics; junior high school

INTRODUCTION

Mathematical procedural skills are essential in mathematics education (Yuswan & Maat, 2021). These skills involve understanding and applying systematic steps or algorithms to solve mathematical problems accurately, effectively, and efficiently (Fatimah & Zakiah, 2018; Oktavianti et al., 2018; Rosdianwinata & Ridwan, 2018; Ulva et al., 2020). Procedural skills enable students not only to grasp theoretical procedures but also to apply them flexibly and appropriately within various contexts, understanding when and how these procedures should be used (Fatimah & Zakiah, 2018; Oktavianti et al., 2018). Developing procedural skills is critical in mathematics because it fosters students' confidence and perseverance in solving diverse mathematical problems, which in turn lays the foundation for conceptual understanding (Antari et al., 2022; Lenamah et al., 2022; Yuliani et al., 2020).

Mastery of procedural skills is a valuable asset for students, as it aids in understanding mathematical concepts and applying them to solve problems effectively. In geometry, procedural skills allow students to follow problem-solving steps sequentially and systematically, leading to accurate solutions (Fatimah & Zakiah, 2018). However, consistently applying these skills can be challenging; students frequently encounter difficulties, such as inadequate concept comprehension, an inability to identify crucial information, and incorrect formula application (Yuliana et al., 2022). These difficulties can lead to common mistakes that obstruct their problem-solving abilities, especially on topics requiring conceptual understanding and procedural precision, such as the Pythagorean Theorem (Rahmawaty & Nurmeidina, 2023). This challenge underscores the need for instructional approaches that foster procedural skill development from an early stage to help students effectively navigate geometric concepts. Enhancing procedural skills is crucial for understanding mathematical concepts and equipping students to tackle more complex challenges in their mathematics education (Pai'pinan & Kho, 2018).

The Pythagorean theorem, a fundamental concept in geometry, is frequently introduced at the junior high school level and holds a central place in the mathematics curriculum. Mastery of this theorem is essential, as it has extensive real-world applications and allows students to engage deeply with procedural skills (Lian et al., 2023; Putra, 2020). Understanding this concept contributes to other mathematical topics, especially those related to the mastery of geometric concepts such as plane figures (Rohimah & Nursuprianah, 2016)). A deep understanding of the Pythagorean Theorem serves as a crucial foundation for solving various mathematical problems related to both plane and solid figures while also reinforcing students' comprehension of other geometric concepts (Muchyidin & Amin, 2012). Furthermore, mastery of the Pythagorean Theorem in mathematics is essential due to its broad applications in everyday contexts, providing students with opportunities to develop procedural skills in depth (Lian et al., 2023; Putra, 2020). Research indicates that students' ability to apply the Pythagorean theorem correctly is closely linked to their understanding of geometric properties and ability to execute mathematical procedures accurately (Al-Mutawah et al., 2019; Gilmore et al., 2017). However, studies also reveal that students frequently need to correct procedural errors, which range from misunderstandings of foundational concepts to incorrect procedural execution (Hatisaru, 2020). These errors highlight gaps in students' procedural skills, which must be identified and analyzed to enable teachers to design targeted interventions to improve these skills (Friantini et al., 2021; Ridwan et al., 2021).

Research on procedural skills in mathematics, particularly at the secondary education level, has garnered significant attention in recent decades. These procedural skills are considered essential in facilitating the understanding of mathematical concepts and problem-solving. Previous studies have shown that students' procedural skills can affect the extent to which they are able to apply mathematical knowledge flexibly, accurately, and efficiently. For example, Fatimah and Zakiah (2018) found that the procedural fluency of high school students greatly influenced their success in solving marketing-related math problems, emphasizing flexibility, accuracy, and efficiency. Furthermore, Ballon et al. (2024) and Faulkner et al. (2023) emphasize the importance of procedural skills for students transitioning to higher education, where procedural skills are often more developed than problem-solving skills, helping them face new challenges at university. Cragg et al. (2017) also showed that procedural skills are strongly related to executive functions, such as working memory, which directly impact mathematical achievement. However, although these studies highlight the importance of procedural skills at different educational levels, most focus on upper-secondary and tertiary students.

At the junior high school level, research on students' procedural skills, particularly in the context of geometry and the application of the Pythagorean Theorem, is still limited. Previous research has primarily identified students' conceptual difficulties in understanding geometry in general. For example, Indrayany and Lestari (2019) observed students' struggles with solving geometry problems related to the area of flat buildings. Similarly, Utami et al. (2017) identified challenges in understanding the concept of rectangles. However, there is a lack of studies specifically addressing the procedural steps students take when solving problems involving the Pythagorean Theorem, which requires a systematic procedural understanding. Additionally, although Puloo et al. (2018) noted differences in the approaches of male and female students in solving geometry problems, the influence of these differences on students' procedural ability to apply the Pythagorean Theorem effectively has not been widely studied.

Based on a review of previous studies, there is a clear gap in the literature regarding the procedural understanding of junior high school students in the context of geometry, especially involving the application of the Pythagorean Theorem. This gap is particularly evident in the lack of in-depth studies on the procedural steps junior high school students take when solving more complex geometry problems, as well as the patterns of procedural errors that often arise during the problem-solving process. Furthermore, data obtained from observations at a junior high school in Kolaka Regency, Southeast Sulawesi, Indonesia suggest that many students still face difficulties in solving mathematical word problems, especially when applying the Pythagorean Theorem. This finding indicates that there are obstacles in mastering procedural skills, which may hinder students' understanding of basic geometric concepts. These challenges require concrete solutions from teachers and researchers to identify strategies that can help students build their procedural skills in solving math problems. As a first step, it is important to understand the current level of students' procedural skills in overcoming geometry problems, as well as the types of errors they frequently make. Therefore, this study aims to fill the gap by exploring the procedural skills of junior high school students in the context of geometry, with a focus on the application of the Pythagorean Theorem.

The novelty of this study lies in its in-depth approach to exploring the procedural skills used by junior high school students when solving geometry problems involving the Pythagorean Theorem. The study also focuses on identifying patterns of procedural errors that frequently arise and examining how obstacles students face can affect systematic and effective problem-solving. Additionally, this study offers insights that can be used to design more effective learning strategies to help junior high school students overcome challenges in applying the Pythagorean Theorem. This approach not only bridges the gap between general procedural skills and the application of the Pythagorean Theorem but also offers practical solutions for overcoming common mistakes and improving problem-solving skills among junior high school students.

This study aims to investigate the procedural skills of junior high school students in solving problems related to the Pythagorean Theorem, to identify the procedural steps they commonly employ, and to propose recommendations for effective instructional strategies. The research endeavours to provide a comprehensive analysis of how junior high school students approach and solve more complex problems involving the Pythagorean Theorem. Hence, the study is guided by the following research questions: (1) What patterns of procedural skills do students exhibit during the problem-solving process? (2) What specific procedural steps are most frequently employed by students in solving problems involving the Pythagorean Theorem? Addressing these questions is expected to deepen the understanding of junior high school students' procedural skills in solving mathematical problems and inform the development of more structured and effective pedagogical approaches.

METHODS

This study employs a qualitative methodology using a case study approach to investigate the mathematical procedural abilities of grade 8 junior high school students in solving problems involving the Pythagorean Theorem. The qualitative approach was selected because it allows for an in-depth exploration of students' procedural competencies. It offers a comprehensive understanding of how they navigate the steps involved in solving geometric problems, specifically those related to the Pythagorean Theorem. This approach aligns with the research objectives of exploring students' procedural skills, identifying the steps they typically take, and proposing recommendations for effective instructional strategies. The study focuses on three key indicators to assess procedural skill stages: (1) understanding problem-solving steps, which includes identifying known information and unknown variables and selecting appropriate methods; (2) correctly implementing procedural steps, with an emphasis on the systematic execution of mathematical operations; and (3) demonstrating flexibility in adapting procedures, including modifying strategies as necessary to achieve accurate solutions (Fatimah & Zakiah, 2018; Pratidiana & Muhayatun, 2021). These indicators provide a framework for categorizing and analyzing both the results of students' problem-solving efforts and their responses during interviews.

The study was conducted with 30 eighth-grade students from a junior high school in Kolaka, Southeast Sulawesi, Indonesia. Three students were selected as research informants to provide in-depth information regarding their procedural skills, representing high, medium, and low levels of mathematical ability. The selection of informants was carried out using purposive sampling based on a combination of the results from problem-solving tasks and teacher recommendations. These recommendations emphasized students' mathematical performance and their communication skills in articulating their reasoning processes. Participants were chosen based on their performance in problem-solving tasks, ensuring a diverse representation of mathematical abilities while also considering the information provided by the teacher. This selection process ensured a representative sample, enabling the exploration of a broad range of procedural skills among the students.

The research instruments employed in this study included a mathematics assignment sheet specifically designed to evaluate students' procedural skills in solving problems related to the Pythagorean Theorem and a semi-structured interview guide. The assignment sheet comprised three essay-type problems that required students to accurately and systematically apply the Pythagorean Theorem to arrive at a solution. These problems were validated by mathematics lecturers and teachers who taught the participants' classes, ensuring their appropriateness and relevance. The problems were presented as follows: *"(1) A staircase leans against a wall and reaches a height of 12 m. If the distance between the wall and the foot of the stairs is 9 m, determine the length of the stairs!, (2) An observer is positioned at the top of a tower with a height of 120 m. The observer sees boat A at a distance of 130 m and boat B at a distance of 150 m. If the base of the tower, boat A, and boat B are aligned, determine the distance between boat A and boat B!, and (3) Two pillars positioned 12 m apart have heights of 22 m and 13 m, respectively. Determine the length of the connecting wire between the tops of the two pillars."*

Semi-structured interviews were conducted to investigate students' strategies, the reasoning behind their chosen procedures, and their understanding of the underlying concepts. Some key questions asked during the interviews included: *"How do you decide which step to use in solving the problem?, Can you explain why you chose this method over others?, How do you ensure that the steps you are taking are correct?, and What challenges did you face while working on the solution?"* These questions were designed to uncover students' strategic thinking, their comprehension of the procedure, and any potential sources of error. The interview questions were developed based on the responses provided by the students during the interview process.

Data collection occurred in several stages. Initially, students individually completed the assignment sheets in class, during which the researchers observed their procedural skills in solving Pythagorean problems. Researchers recorded the students' behaviours, such as the strategies they employed to solve the problems. After the assignment, students' responses were carefully analyzed to assess their procedural skills. In addition, semi-structured interviews were conducted to gain further insight into the students' problem-solving processes, the procedural skills they applied, and the rationale behind their decision-making. All interviews were recorded to ensure data accuracy and to facilitate detailed analysis.

The collected data was analyzed in several stages, involving data presentation, reduction, interpretation, and conclusion drawing (Miles et al., 2014). The data presentation stage involved categorizing the results of students' problem-solving efforts and interview responses according to the indicators of mathematical procedural skills. Data reduction focused on selecting and simplifying information relevant to the research questions. The interpretation stage synthesized the findings into a coherent narrative, highlighting procedural errors and areas requiring improvement. Finally, conclusions were drawn by analyzing the relationships between categories and proposing strategies to enhance students' procedural fluency. Data triangulation was employed by cross-verifying information from task sheets, interview transcripts, and observational field notes to ensure the consistency and validity of the findings.

RESULT

Patterns of Procedural Errors in Solving Pythagorean Theorem Problems

Analysis of procedural skill patterns across different proficiency levels reveals distinct trends, with each group encountering unique challenges in their problem-solving steps, particularly in the application of the Pythagorean Theorem. These findings highlight how varying levels of mathematical ability not only influence procedural knowledge and the accuracy of answers but also affect the efficiency of mathematical problem-solving. The group of students with high procedural skills (S-01) demonstrated proficiency in problem-solving, primarily characterized by structured and systematic resolution procedures. This proficiency is exemplified by the solution provided by S-01, as shown in Figure 1.

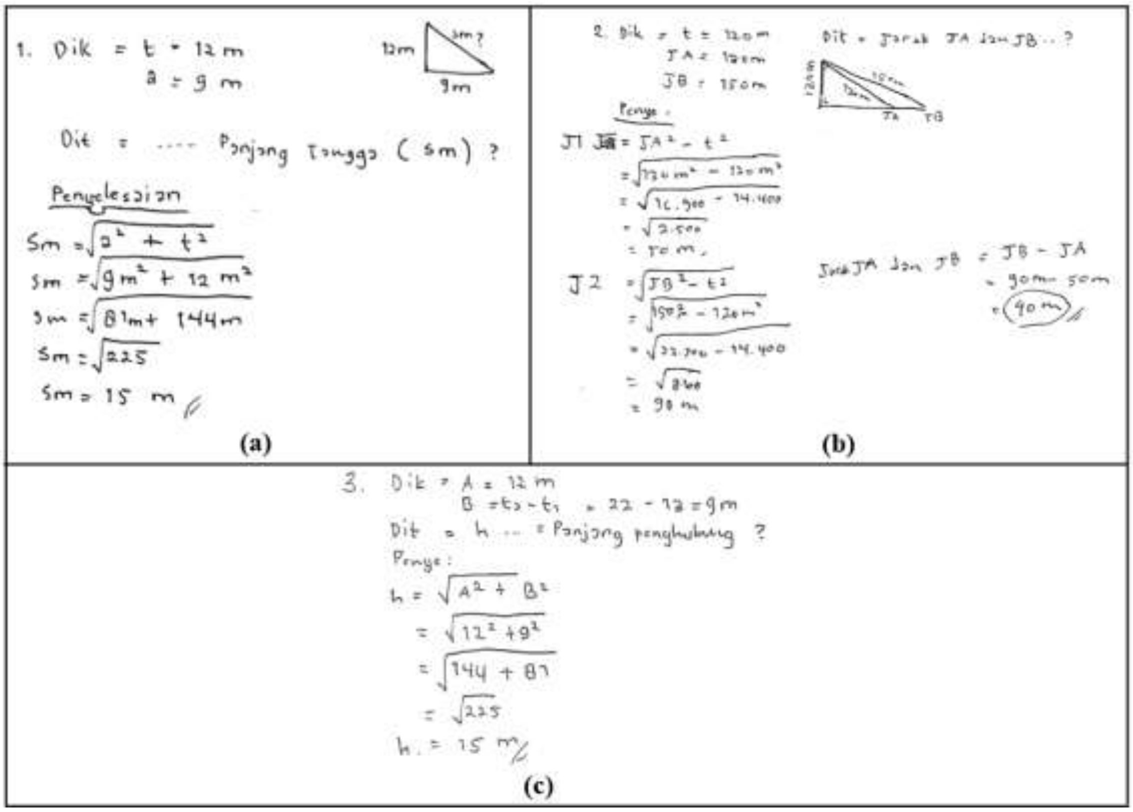

Figure 1.
Student Answer S-01

Based on Figure 1 (a), it can be observed that S-01 effectively utilizes their procedural skills in solving the first problem. S-01 demonstrates a clear understanding of the problem by presenting an illustration of a right triangle, where the length of the ladder is identified as the hypotenuse. Furthermore, S-01 follows a structured approach to problem-solving, applying the Pythagorean Theorem methodically. In the second and third problems, depicted in Figure 1 (b) and Figure 1 (c), S-01 consistently adheres to the problem-solving steps by first recording the given information, the objectives to be achieved, and the steps to solve the problem. All the steps are written correctly and in a structured manner, with each result accompanied by the appropriate units of measurement. Interviews with S-01 revealed a strong awareness of the problem-solving process, as the student expressed, *"In the process of solving the problem, I try to double-check my steps to ensure that my* *resolution process is correct and free of errors."* This statement reflects the student's high level of procedural skill, demonstrating a careful review of each step to avoid mistakes in the final solution.

In the middle-ability group (S-02), error patterns are observed frequently, primarily due to inconsistencies in applying procedural skills. Additionally, some errors stem from a limited understanding of mathematical concepts, particularly in interpreting written expressions or illustrative representations. These issues are evident in the problem-solving steps of S-02, as depicted in Figure 2.

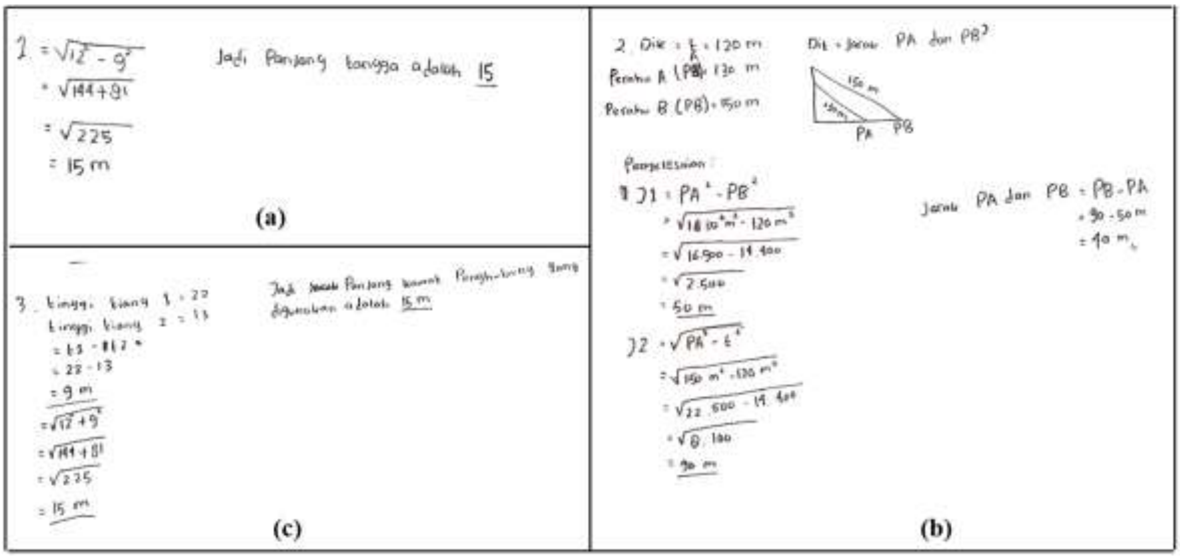

Figure 2.
Student Answer S-02

In Figure 2 (a), it is observed that while solving the first problem, S-02 began the solution process without first writing down the known information. He started by writing the symbol "=" and then proceeded to write the mathematical expression, which involved the square root of the subtraction of two squared numbers. In this step, S-02 made an error by using the subtraction symbol instead of the addition symbol. However, in the subsequent step, he correctly applied the addition symbol. Additionally, S-02 omitted the unit of length in his conclusion, even though the value he found was correct. In the second problem, shown in Figure 2 (b), S-02 makes an error in the illustration, representing the two triangles as intersecting only at one corner. The correct illustration should depict the two triangles intersecting at both the right angle and the top corner of the triangle, which corresponds to the top of the tower. Despite this error in the diagram, the calculation process remains accurate. In the third problem, as seen in Figure 2 (c), S-02 does not clearly define the variables on the left side of the sign "=". After noting the heights of the two poles, S-02 proceeds directly to write the operation symbol and its corresponding expression. While the final solution is correct and includes the proper unit of measurement, more clarity in defining the sought value in the solution step is needed. Interviews with S-02 revealed that *"I often pay less attention to what I have written or illustrated, and only focus on the ideas and final goals that I want to achieve."* This behaviour contributes to inconsistencies in the answers provided, In the group of students with lower abilities (S-03), the pattern of procedural skills is characterized by frequent errors in the solution steps, which result from an unstructured approach to problem-solving. The responses provided by S-03 are shown in Figure 3.

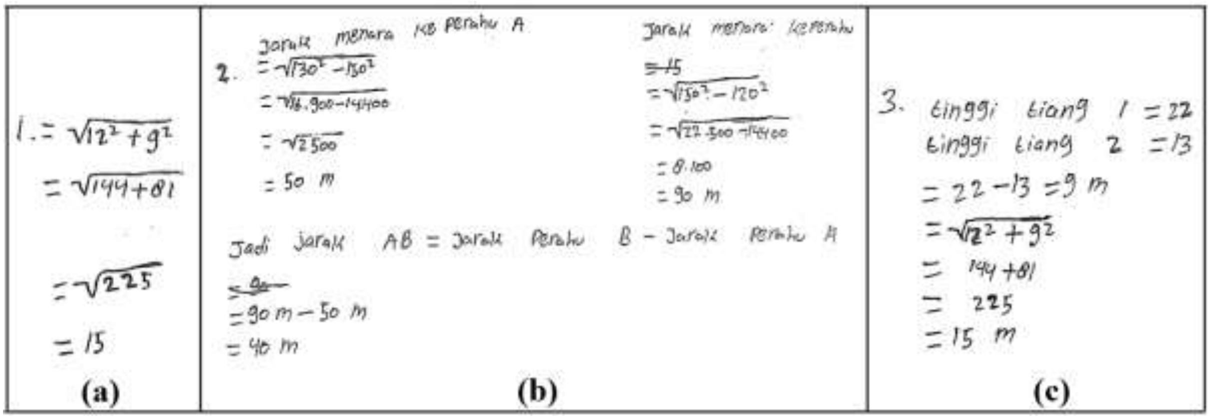

Figure 3.
Student Answer S-03

Based on the observations in Figure 3, it was found that S-03 outlined the steps for solving mathematical problems in a disorganized manner. In the first problem (Figure 3a), S-03 began by writing the symbol "=", followed by the continuation of the solution. However, he stopped after determining the value "15," failing to explain its meaning or include the appropriate unit of measurement. In the second problem (Figure 3b), S-03 presented the solution steps similarly, though he clearly identified the value to be determined, specifically "the distance from the tower to the boat." Despite this, S-03 made a procedural error in his mathematical expression by incorrectly writing √8100, omitting the square root symbol. In the third problem (Figure 3c), S-03 repeated the same error, incorrectly writing the expression "144+81" rather than applying the square root symbol (√). Although the final solution was correct, the procedural steps contained significant errors. Interviews with S-03 indicated that he frequently experiences difficulty in analyzing mathematical problems, particularly those presented in word problem format. He explained, "I am often confused in analyzing math problems, especially word problems. For that, I usually use trial and error to solve math problems, especially in parts that are difficult for me to understand." Additionally, S-03 acknowledged that he rarely revisits or checks his work, which leads to frequent errors in his written answers due to carelessness and neglect in the re-checking phase.

Overall, these findings highlight variations in students' procedural skills according to their mathematical ability levels. The results underscore the importance of encouraging students to adopt a structured, step-by-step approach when solving mathematical problems, particularly those involving the Pythagorean theorem. The differences in error patterns across skill levels emphasize the critical role that procedural skills play in enhancing both accuracy and confidence in mathematical tasks. These findings suggest the need for instructional strategies that focus on systematic problem-solving, particularly for students with moderate to low mathematics abilities. Such strategies should be implemented to improve accuracy and reduce errors related to the omission of steps or misinterpretation of relevant underlying concepts.

Typical Procedural Steps Carried Out by Eighth-Grade Students in Solving Pythagorean Theorem Problem

Based on the analysis of procedural skills, it was observed that the steps performed by eighth-grade students varied significantly according to their proficiency levels. These variations were categorized into high (S-01), medium (S-02), and low (S-03) skill levels based on students' ability to understand, apply, and execute problem-solving steps. Observations and responses from student interviews corroborated these differences in procedural skills.

Students with high ability category (S-01) demonstrated a structured approach to solving Pythagorean theorem problems. They consistently outlined each step, first identifying known elements and unknown variables, then methodically working through each part of the solution. According to the interview results, student S-01 explained, "I always start by writing down what I know and what I need to know; it helps me stay focused on each step." This approach aligns with problem-solving frameworks that emphasize systematic planning as the foundation for accuracy in mathematical reasoning. Observational data further indicated that S-01 students frequently reviewed their steps after completing the solution to ensure accuracy, suggesting a heightened awareness of potential miscalculations or misinterpretations during the process.

In contrast, students in the medium ability category (S-02) showed inconsistencies in their approach. Although they sometimes listed known and unknown information, they often skipped structured steps and proceeded directly to finding a solution. Interviews with students in this group revealed a tendency to skip detailed steps because they believed they could visualize the solution. Student S-02 explained, “I usually think I know where the problem is going, so I don’t always write it all down.” However, this approach often led to skipped steps or premature conclusions, especially in more complex problems that required precision. Additionally, they sometimes lacked thoroughness in understanding a problem, which contributed to errors in certain steps of the solution, as well as incompleteness in writing down the units of certain quantities.

For students with low ability (S-03), the approach was very different. These students typically omitted procedural steps altogether, jumping straight to what they perceived as the solution phase. They needed to show more understanding of the need for a structured approach, often skipping critical steps that would clarify the solution path. Interview insights from this group highlighted a reluctance to engage with problems systematically. Student S-03 stated, “I just tried to solve it right away because writing down the steps felt like it was slowing me down.” This tendency to bypass structured reasoning not only led to frequent errors but also indicated a misunderstanding of the role of systematic problem-solving in mathematics. Based on the findings described, the characteristics of students’ procedural skills in solving Pythagorean problems can be summarized in Table 1.

Table 1.
Characteristics of Student Procedural Skills

Indicator	Characteristic		
	S-01 (High)	**S-02 (Medium)**	**S-03 (Low)**
Understanding Procedures	The student consistently records problem-solving steps in a structured manner, listing known information,	The student is inconsistent in recording problem-solving steps in a structured manner; in some cases, they proceed directly to the solution stage without clearly	The student does not record solution steps in a structured manner, moving directly to the solution stage without explanations or details.

	questions, and solution steps.	writing the details of the given values.	
Understanding the Use of Procedures	The student accurately understands the application of problem-solving procedures and applies them correctly.	The student occasionally understands the application of problem-solving procedures but makes errors in illustrating problems and is inconsistent in writing units for quantities.	The student is inconsistent in understanding the application of problem-solving procedures, with numerous errors in writing or applying procedures.
Skill in Executing Procedures	The student is skilled in executing procedures flexibly, accurately, and efficiently, with minimal errors.	The student demonstrates flexibility, accuracy, and efficiency in executing procedures, though inconsistently, and may require assistance.	The student is not yet skilled in executing procedures flexibly, accurately, or efficiently, with numerous execution errors.

DISCUSSION

Patterns of Procedural Errors in Solving Pythagorean Theorem Problems

The research findings reveal patterns in procedural skills that vary according to the level of student ability. Students with high mathematical ability tend to demonstrate a high level of accuracy in applying procedures and do not make mistakes in their problem-solving steps. This finding is consistent with the work of Sasangka et al. (2022), Lutfiya et al. (2021) and Alawiyah et al. (2024), which indicate that students with high ability show high accuracy in mathematical problem-solving procedures. Additionally, students with high ability exhibit strong procedural skills, demonstrating a deep understanding of the mathematical concepts they have mastered. They can apply these concepts accurately, efficiently, and flexibly to the problems presented (Fatimah & Zakiah, 2018; Pratidiana & Muhayatun, 2021). High-ability students also exhibit the behavioural pattern of consistently checking their solution steps, which aligns with the findings of Samosir and Herman (2024), who noted that students with high mathematical achievement consistently follow Polya's problem-solving stages, including the step of reviewing the solution.

In contrast, students with moderate abilities tend to make mistakes in the form of inconsistencies when applying systematic problem-solving steps and demonstrate limitations in illustrating related geometric concepts. This finding is supported by Ayuningtyas et al. (2021), who noted that students with moderate abilities often make procedural errors due to inconsistency in applying problem-solving steps. Similarly, Devi and Amir (2021) found that students with moderate mathematical abilities frequently struggle with mathematical problems, exhibiting inconsistent step application and limitations in systematically illustrating geometric concepts. Furthermore, students in the moderate ability group showed limited mastery of other procedural skills, such as failing to re-check the problem-solving steps they had taken. This observation aligns with Samosir and Herman (2024), who found that moderate-achieving students typically only engage in the stages of understanding the problem and implementing a plan, often neglecting the reflection stage. The inconsistency in applying systematic steps and the absence of a reflection process in the moderate-ability group highlight the need for additional instructional strategies tailored to this group.

Students with low mathematical abilities exhibit procedural skills that require significant improvement. These students often make prominent errors, such as skipping important steps and making mistakes in writing mathematical expressions. This finding aligns with Sasangka et al. (2022) and Wati and Ningtyas (2020), who observed that students in the low mathematical ability category commonly neglect procedural steps and make errors in mathematical expression writing. Such errors suggest that these students lack strong procedural skills, particularly in structuring solution steps and ensuring accuracy in writing mathematical expressions. Moreover, students in this group need help to demonstrate flexibility and accuracy in executing mathematical procedures, which negatively impacts the final accuracy of their problem-solving results (Fatimah & Zakiah, 2018). This finding is consistent with Al-Mutawah et al. (2019), who reported that students with low procedural skills often fail to recognize the critical importance of procedural structures in solving complex mathematical problems.

The variation in error patterns observed in this research highlights the crucial role procedural skills play in determining the accuracy and efficiency of students' solutions to mathematical problems. As Gilmore et al. (2017) noted, procedural skills encompass not only technical abilities in performing calculations but also adequate planning, execution, and review at each step of the solution. The results of this study underscore the importance of instructional strategies that prioritize the development of procedural skills through structured practice and a deep understanding of procedural application, especially for students in the medium and low ability categories.

Typical Procedural Steps Carried Out by Eighth-Grade Students in Solving Pythagorean Theorem Problem

Research findings show that students with high levels of mathematical ability consistently follow a structured approach to problem-solving. This aligns with indicators of procedural skills, which involve the ability to understand and apply systematic steps in solving problems accurately and efficiently (Fatimah & Zakiah, 2018; Oktavianti et al., 2018; Pratidiana & Muhayatun, 2021; Ulva et al., 2020). These findings suggest that students with high mathematical ability possess strong procedural skills, enabling them to identify and record important elements at each stage of problem-solving. This skill set supports the accuracy of the resulting solution. Additionally, this ability is consistent with the work of Oktavianti et al. (2018) and Pratidiana and Muhayatun (2021), who found that students with strong procedural skills are able to apply mathematical procedures flexibly according to the context, while being mindful of the process.

In the group of students with moderate mathematical abilities, there was inconsistency in their mathematical procedural skills when solving Pythagorean theorem problems. The research findings indicate that, although these students have a basic understanding of the importance of a procedural approach, they are often inconsistent and tend to skip initial steps, such as writing down known information and identifying the targets sought. Felia and Defitriani (2021) observed that students with moderate abilities often exhibit inconsistent procedural skills by skipping initial steps in solving problems that are considered simpler. This tendency suggests that students have a limited understanding of deep procedural skills. While they may grasp the steps to solve a problem to some extent, they have not yet developed the ability to apply these steps consistently. This finding aligns with research indicating that students with partial procedural understanding often encounter difficulties in building systematic structures for more complex mathematical tasks (Ulva et al., 2020; Yuliana et al., 2022).

Meanwhile, students with low levels of mathematical ability struggle with using procedural skills to solve Pythagorean problems. Their lack of understanding of the importance of recording information and following systematic steps is a significant barrier to their conceptual understanding. This finding is consistent with Ruhma and Madawistama (2023), who noted that students with low abilities face challenges in applying mathematical concepts contextually and in breaking down complex information into manageable stages. These students often find themselves struggling to understand and solve the problems presented (Rahmawati et al., 2022). Additionally, the research findings show that students with low abilities have not yet developed the flexibility or accuracy needed to apply procedural steps effectively, resulting in a higher likelihood of incorrect answers. Pai'pinan and Kho (2018) emphasize that the inability to follow procedural steps systematically is closely linked to students' lack of skills in comprehensive mathematical planning and reasoning.

In general, the variation in the application of procedural steps among the three groups of students highlights the importance of structure and planning in solving mathematical problems. This finding aligns with Antari et al. (2022), who noted that procedural skills are not only related to understanding the steps for solving mathematical problems, but also to how students internalize these procedures across different problem contexts. Therefore, it is crucial to develop instructional strategies that emphasize a structured procedural approach, enabling students with medium and low mathematical abilities to build a stronger procedural understanding.

Instructional Implications and Recommendations

The findings of this study indicate that mathematical procedural skills in applying the Pythagorean Theorem are essential for enhancing the accuracy and efficiency of students' responses. This statement aligns with Sari et al. (2020), who found that many students experience conceptual and procedural errors and computational inaccuracies when solving Pythagorean problems. Based on these findings, teachers should consider integrating research insights into their instructional design. For example, adopting scaffolding strategies tailored to students' ability levels can be effective (Rahmawati et al., 2022; Ruhma & Madawistama, 2023). For students with medium abilities, teachers can incorporate guided practice sessions, while for students with low abilities, targeted reinforcement of basic steps is essential.

Students with low mathematical abilities tend to make errors in their answers due to a limited understanding of the written procedures. This lack of procedural literacy can hinder their development of broader conceptual understanding. Mastery of mathematical procedural skills is foundational for developing critical and analytical thinking, which are essential for success in advanced mathematics (Antari et al., 2022; Fatimah & Zakiah, 2018). As Sari et al. (2020) and Permatahati et al. (2022) state, critical thinking skills are closely related to both conceptual and procedural understanding in mathematical performance. Therefore, it is important to emphasize the development of procedural skills in students with low abilities, with an early introduction of these skills in the mathematics curriculum.

To support students with moderate and low mathematical abilities, instructional approaches that strengthen procedural skills are essential. One strategy that can be applied is the integration of conceptual and procedural knowledge (Yuswan & Maat, 2021). Additionally, the scaffolding approach proposed by Buaddin Hasan (2020) can be particularly effective for students with low skills. This approach provides gradual guidance, helping students achieve independence in following correct procedures. Rachmawati (2019) and Mardaleni et al. (2018) further support this, noting that scaffolding can improve students' mathematical problem-solving abilities and promote learning independence. Furthermore, relevant contextual problem-based exercises are recommended to help students understand the practical applications of the procedures they learn (Jatisunda, 2016; Sa'diah & Nahdi, 2023). Through these strategies, students are expected to improve both their flexibility and accuracy in applying procedural steps.

CONCLUSION AND IMPLICATION

Conclusion

This study reveals variations in the mathematical procedural skills of eighth-grade students when solving Pythagorean Theorem problems, with differences largely influenced by students' mathematical ability levels. In general, students with high mathematical ability demonstrate strong procedural skills, with a deep understanding of the problem-solving process and the ability to apply steps systematically and accurately. These students not only record known information and solution steps in detail but also show a consistent awareness of the need to re-verify their work to ensure the accuracy of the final results. This behaviour aligns with the key characteristics of procedural skills, where accuracy, flexibility, and efficiency in executing procedures are crucial indicators of successful problem-solving.

In the group of students with moderate ability, procedural skills tend to be inconsistent. These students often skip important initial steps in the problem-solving process. While they have a basic understanding of the procedural approach, they frequently neglect the planning stage and tend to bypass the step of re-checking their answers. As a result, students with moderate abilities sometimes make errors in recording known information or illustrating solution steps, particularly in more complex problems. This act suggests that although these students grasp the basic concept of procedures, they struggle to apply the steps consistently and comprehensively, leading to suboptimal accuracy in their problem-solving.

Meanwhile, students with low mathematical abilities demonstrate low procedural skills, with a limited understanding of the need for systematic and structured steps in problem-solving. This group often skips the planning stage and moves directly to solving the problem without properly noting or observing each necessary step. This neglect leads to frequent procedural errors, including mistakes in writing mathematical expressions, misunderstanding the problem-solving structure, and inconsistency in noting the units of a quantity. Additionally, students with low abilities engage minimally in verifying their answers, making them more prone to calculation errors.

Overall, this study underscores the importance of a structured procedural approach in helping students develop accurate and efficient mathematical problem-solving skills. To improve procedural skills, teachers should design learning activities with tailored interventions for each ability level. For students with low ability, structured practice and reinforcement of basic procedures can help them understand the significance of each step in the problem-solving process. For students with intermediate ability, a practice that emphasizes consistency in applying procedures, along with a focus on verification, can reduce errors and improve accuracy. For students with high ability, the emphasis should be on refining the detail and accuracy of calculations, ensuring they maintain high precision when solving more complex problems. Understanding students' procedural skills in solving the Pythagorean Theorem, based on their ability levels, can provide valuable insights for teachers in developing guided mathematics learning modules and designing activities that support the development of higher-order thinking skills (HOTS) in junior high school students.

However, this study has limitations that should be considered when interpreting the results. One limitation is the small sample size, which consists of only three students with varying ability levels from a single school. As a result, these findings may need to be more generalizable to the broader student population. Additionally, the focus of this study on a single mathematical concept—the Pythagorean Theorem—may not fully capture the variation of procedural skills across other mathematical topics. While the qualitative approach provided in-depth insights, it has limitations in terms of generalizability. Therefore, further research with a larger sample size and a broader range of mathematical contexts is recommended to confirm the consistency of these findings related to procedural skills. A quantitative approach could also offer more representative data and stronger generalizations. Further research could explore instructional strategies that leverage the findings of this study, with a particular focus on strengthening students' procedural skills through scaffolding approaches and guided practice. The implementation of these strategies is expected to offer practical guidance for improving procedural skills at various educational levels and across different mathematical topics.

Implication

This research highlights the need for targeted instructional strategies to strengthen students' mathematical procedural skills according to their ability levels. For low-skill students, structured exercises on basic procedural steps can enhance comprehension and accuracy; moderate-skill students may benefit from focusing on procedural consistency and verification to reduce errors, while high-skill students can improve through precision-focused practices. Emphasizing procedural fluency early in the curriculum improves problem-solving accuracy and fosters critical thinking essential for advanced mathematics. Integrating scaffolding and contextual problem-based learning can further support these skill developments effectively.

Disclosure statement

The author(s) declare that there are no conflicts of interest related to this manuscript's research, authorship, and publication. This study was conducted independently without any financial or personal relationships that could influence the results and interpretations presented in this work.

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